An explanation for Ohm's law

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The phenomenon that the electrical resistivity, defined as the quotient of the electric field and the electric current density, is independent of the electric field is called Ohm's law. I show how this can be explained with a new interpretation of the relaxation time used in the Drude model.

The electrical resistivity can be described by the Drude model with the formula

$$\rho = \frac{m}{ne^2\tau}, \tag{1}$$

where m is the mass and e the charge of an electron, n is the density of electrons, which are constants, and τ the relaxation time. In it the relaxation time is interpreted as the period of time in which an electron loses its electric excitation energy, i.e., its drift velocity becomes zero, whereas its thermal excitation energy is assumed to determine the magnitude of the relaxation time. But to my mind, it is reasonable to include all excitation energies, i.e., the relaxation time is the period of time in which an electron loses its total excitation energy which thereby determines the magnitude of the relaxation time. In general the total excitation energy constitutes of the electric and the thermal excitation energy,

$$E = E_{el} + E_{therm}. \tag{2}$$

The orders of magnitude can be estimated with the Fermi model

$$\begin{aligned} E_{el} &= \frac{1}{2}m\left(\mathbf{v}_F + \mathbf{v}_D\right)^2 - \frac{1}{2}m\mathbf{v}_F^2 \\ &= \frac{1}{2}m\left(2v_F v_D \cos\gamma + v_D^2\right) \\ &\approx m v_F v_D, \end{aligned} \tag{3}$$

where v_F is the Fermi and v_D the drift velocity, and $v_F \gg v_D$ and $\cos\gamma = 1$ is used, and with the equipartition relation

$$E_{therm} \approx k_B T. \tag{4}$$

Under typical conditions one finds $v_F \approx 10^6\,\mathrm{ms}^{-1}$, $v_D \approx 10^{-3}\,\mathrm{ms}^{-1}$, and $T \approx 300\,\mathrm{K}$, which means that

$$E_{el} \approx 10\,\mathrm{neV}, \tag{5}$$
$$E_{therm} \approx 10\,\mathrm{meV}. \tag{6}$$

Hence, we have $E_{therm} \gg E_{el}$ which implies that $E = E_{therm} = \mathrm{const}$ and $\tau(E) = \mathrm{const}$ which again implies that $\rho = \mathrm{const}$ which is the statement of Ohm's law. Furthermore, the new point of view predicts the limit $E_{therm} \gg E_{el}$ for the validity of Ohm's law, i.e., as soon as E_{el} reaches the order of magnitude of E_{therm}, by large drift velocities or small temperatures, the electrical resistivity starts to depend on the electric field.

Just as well, of course, we can look at the equation of motion of an electron. In the Drude picture one finds

$$m\frac{dv}{dt} = -e\mathcal{E} - m\frac{v_D}{\tau}, \tag{7}$$

where $\mathcal{E}$ is the applied electric field which accelerates the electron and the second term describes the resistance against the acceleration. For the case that the electric field is switched off ($\mathcal{E} = 0$), one obtains so

$$v(t) = v_D\left(1 - \frac{t}{\tau}\right), \tag{8}$$

which means that τ is the period of time in which the drift velocity becomes zero. In my new picture, though, I say that Eq. (7) is incomplete and must be completed by the acceleration resulting from the thermal excitation energy. In normal metals, for example, electrons interact with thermally excited phonons and get in this way accelerated. The corresponding force I formulate in analogy to the resistance

$$\frac{E_{therm}}{v\tau} = \frac{k_B T}{v_F \tau}, \tag{9}$$

where the energy (4) is gained in the period of time τ, and thus in the distance $v\tau$. The introduced total velocity v constitutes of the Fermi velocity v_F, the thermal velocity v_{therm}, and the drift velocity v_D, where typical values for the velocities are $v_F \approx 10^6\,\mathrm{ms}^{-1}$, $v_{therm} \approx 10^4\,\mathrm{ms}^{-1}$, and $v_D \approx 10^{-3}\,\mathrm{ms}^{-1}$ which means that $v_F \gg v_{therm}, v_D$ which is why the total velocity is set equal to the Fermi velocity. Now, using the isotropy of the thermal acceleration, Eq. (7) becomes

$$m\frac{dv}{dt} = -e\mathcal{E} \pm \frac{k_B T}{v_F \tau} - m\frac{v_0}{\tau}, \tag{10}$$

where the resistance is adjusted to the new situation. In it v_D becomes v_0 which constitutes of both v_D and v_{therm}, and τ becomes the period of time in which v_0 becomes zero. For the steady case ($dv/dt = 0$) I get then

$$v_0 = -\frac{e\tau}{m}\mathcal{E} \pm \frac{k_B T}{m v_F} \quad (11)$$

$$= v_D + v_{therm}. \quad (12)$$

Hence, just as for the above considerations, because $v_{therm} \gg v_D$, or in fact $|v_{therm}| \gg |v_D|$, and thus v_{therm} dominates the magnitude of the relaxation time, one obtains $\tau = \tau(v_0) = \tau(v_{therm}) = \tau(T)$ and the independence of τ from $\mathcal{E}$, which is Ohm's law.

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