

Solving the mystery of superconductivity by equating two equations

T. Buttler*
Munich, Germany
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The mystery of understanding superconductivity, which kept physicists busy over the last 100 years, is considered to be solved by equating two equations. First, one takes Heisenberg's uncertainty relation $t(E) = 2\hbar/E$. Second, one takes the temperature-dependent relaxation time found in the Drude formula for the electrical resistivity $t(T) = m/\rho(T)ne^2$, where the temperature T is identified with the excitation energy E via the equipartition relation. By means of the quantities concerning time, energy, space, and momentum obtained from the equating all major phenomena connected with superconductivity, such as, the perfect conductivity, the energy gap, the specific heat, the type I and the type II superconductors, the isotope effect, and the emergence of high-temperature superconductivity can be explained. As one example, one obtains the isotope effect $T_c \propto \Theta^{5/4}$ if the electrical resistivity is described by the Bloch-Grüneisen law $\rho(T) \propto (T/\Theta)^5$ and T is identified with T_c . To make the most use of them the quantities must be interpreted correctly. This matter is tackled by formulating a new postulate. In it the quantities are labeled with the expression "virtual," e.g., the virtual relaxation time and the virtual excitation energy. And its application to solid-state physics yields descriptions which are illustrative, simple, precise, and complete in a way which is not reached by any theory of superconductivity. This is why it seems reasonable that this is the door that needed to be opened to reveal the heart of superconductivity and by that to be able to understand superconductivity. In addition, the postulate yields fundamental insights into the nature of interactions. Therefore, if the presented views turn out to be true, this theoretical work improves our understanding not only of superconductivity, but of physics in general on a grand scale.

I. INTRODUCTION

If one decides to create a theoretical work in physics, one should make oneself clear that there are two approaches to tackle the challenge. Firstly, one postulates a new concept, e.g., a mathematical equation or a condition for a physical quantity, which is not questioned. Secondly, one takes an existing concept and uses it in a new way. In principle, the second approach is preferred because it is an endeavor of physicists to describe nature with the least necessary number of concepts. Nevertheless, as the past has shown, from time to time the introduction of a new concept is unavoidable to be able to explain certain phenomena. The last great new concepts were the quantum physical equations and the constancy of the speed of light postulated at the turn of the nineteenth century by, among others, Einstein, Planck, Schrödinger, and Heisenberg. The question that opens up now is which of the two approaches has to be used to explain superconductivity, which is the challenge tackled within this work. I believe that the first one has to be used which is why a new concept in form of a new postulate is introduced. A circumstance that supports the choice is that physicists had great difficulties with finding an explanation with the existing concepts (there exists the large period of time of nearly 50 years between the experimental observation in 1911 by Kamerlingh Onnes¹ and the theoretical explanation in 1957 by Bardeen, Cooper, and Schrieffer²), and with the discovery of high-temperature superconductivity in 1986 by Bednorz and Müller,³ it turned out that the found explanation does not cover the phenomenology of superconductivity by far. This situation implies that a new

concept has to be introduced and that the heart of superconductivity is still not revealed.⁴ The statement by Feynman⁵ "Quantum mechanics was developed in 1926, and in the following decade it was rapidly applied to all kinds of phenomena with an enormous qualitative success. The theories of metals and other solids, liquids, chemistry, etc. came out very well. But as we continued to advance the frontiers of knowledge, we left behind two cities under siege which were completely surrounded by knowledge although they themselves remained isolated and unassailable," where the two cities were superfluidity and superconductivity, describes the same difficulty. Thus, to my mind, the reason for "leaving the city of superconductivity behind" is that quantum mechanics is not able to explain superconductivity (in analogy to the reason why classical physics is not able to explain the radiation spectrum of a black body). Next, the ideas behind the new concept are presented.

Its core is the equating of Heisenberg's uncertainty relation

$$t(E) = \frac{2\hbar}{E},$$

and the excitation-energy-dependent relaxation time found in the Drude formula for the electrical resistivity

$$t(E) = \frac{m}{\rho(E)ne^2}.$$

By that one gets two equations each with two unknown quantities which means that by equating them one obtains E_0 and t_0 . Before continuing with my interpretation, I say two things. Firstly, in all literature I came

across such an equating was not found which makes it reasonable to suppose that the approach is new. Secondly, I am convinced that it is worthwhile to concern oneself with it and that it is a promising instrument to solve physical problems. Next, my interpretation is presented. The starting point is that it is assumed that there exists a lower limit for the excitation energy of a particle in certain systems (in analogy to the upper limit for the velocity given by the speed of light). The minimum excitation energy E_0 is a feature of nature which is not questioned. The magnitude of E_0 is determined by equating Heisenberg's uncertainty relation and the excitation-energy-dependent relaxation time. On the one hand, Heisenberg's uncertainty relation is chosen because it is often used to explain minimum energies. They are explained by an argument like "for the period of time $t = 2\hbar/E$ the conservation of energy can be violated and a particle is able to gain the energy E ." On the other hand, it is obvious to choose the excitation-energy-dependent relaxation time because it describes the mean period of time within which a particle loses its excitation energy. And, in an equilibrium, the expressions $t = 2\hbar/E$ and $t = m/\rho mc^2$ are equated. The question that opens up next is how a particle is able to exhibit a minimum excitation energy. In the picture developed so far⁶ the excitation energy is in general given by the thermal energy which is given by the equipartition relation

$$E = \frac{3}{2}k_B T,$$

if one looks at a particle with three translational degrees of freedom and quantum effects are neglected. Thus, the minimum energy E_0 is also identified with a minimum temperature T_0 . For temperatures $T > T_0$ no new effects occur because the thermal exceeds the minimum energy. However, for temperatures $T < T_0$ there must exist a mechanism responsible for the constancy of E_0 which looks as follows: A particle exhibits an additional excitation energy which is called "virtual" excitation energy E_v . The total excitation energy constitutes of the virtual and the thermal energy and equals E_0 . Therefore E_v increases with decreasing T because then the contribution of the thermal energy to E_0 decreases which means that the contribution of the virtual energy has to increase. And, finally, at $T = 0$ K, one gets $E_v = E_0$. Furthermore, it is assumed that the virtual energy has a second function. Namely, it is responsible for the emergence of a new interaction. Its appearance is that a particle performs a virtual relaxation process in which the energy E_v is lost within the period of time t_v . And, simultaneously, a virtual momentum p_v is lost within the distance x_v . By that one has the characteristic quantities which describe the interaction. By performing the relaxation process, a particle gives off a part of E_v which is called Δ_v . Thus, by interacting with each other, particles exhibit an energy which is lower than E_0 by Δ_v , i.e., $E_0 - \Delta_v$ instead of E_0 . The quantity Δ_v is treated

in a related way to E_v which means that Δ_v is identified with a temperature Υ_v and, at $T = 0$ K, one gets $\Delta_v = \Delta_0$ and $\Upsilon_v = \Upsilon_0$. In Fig. 1 these considerations are illustrated.

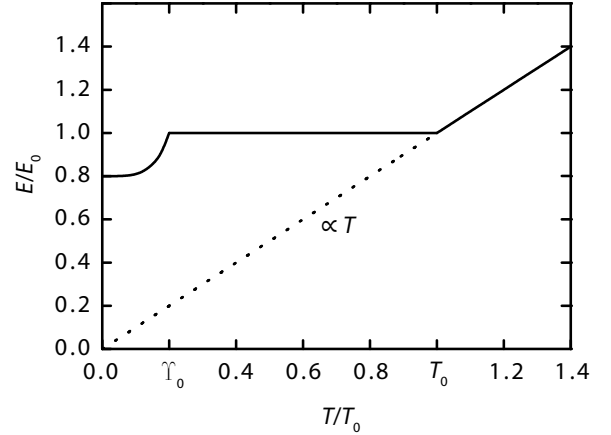


FIG. 1: Excitation energy E of a particle to its minimum excitation energy E_0 vs T/T_0 for $E_0/\Delta_0 = T_0/\Upsilon_0 = 5$. The new concept predicts that there are systems which exhibit the lower limit E_0 for E , whereas the picture developed so far predicts that E is proportional to T for all temperatures (dashed line). Furthermore, by interacting with each other, particles give off the energy Δ_v at $T < \Upsilon_0$.

Next, I make some remarks. A fact that goes with the first approach is that one enters unknown territory. This makes it challenging to elaborate a work which exhibits no false views. I struggled with these difficulties. This is why it is conceivable that some elaborated views need to be improved. Hence, the statement "solving the mystery of superconductivity" in the title of the article is meant to describe that an approach is presented which has the potential to be the door that needed to be opened to reveal the heart of superconductivity. And, I could not withstand to formulate a title which implies that the mystery of superconductivity, which kept physicists busy over the last 100 years and which almost every great physicist has tried and failed to solve, can be solved by an easy approach of "equating two equations." Finally, only time tells if the situation is that astounding. In the concluding paragraph the organization of the article is discussed.

In Sec. II the postulate is formulated and Sec. III contains its interpretation. In Secs. IV and V it is applied to solid-state physics and thereby to superconductivity. Whereas Sec. IV is concerned with the virtual energies, and in case of the antiferromagnetic lattice with the excitation-energy-dependent relaxation time, Sec. V deals with all major phenomena connected with superconductivity except the Josephson effect. For the reader who wants to glance at the results in Sec. V, I point out that the critical temperature T_c is called Υ_0 and the energy gap Δ at $T = 0$ K is called Δ_0 [$T_c = \Upsilon_0$ and $\Delta(T = 0 \text{ K}) = \Delta_0$]. At last, I say something about Appendix B. As the reader may have noticed, the

excitation-energy-dependent relaxation time is set equal to the relaxation time in the Drude formula despite of that it is interpreted differently. However, it turns out that if it is interpreted as an excitation-energy-dependent relaxation time new insights to the electrical resistivity follow, such as, an explanation for Ohm's law and a new limit for its range of validity, presented in Appendix B.

II. THE POSTULATE

The following postulate is made:

There exists a second form of excitation energy besides the thermal excitation energy. The new quantity is called the “virtual” excitation energy E_v . A particle exhibits a virtual excitation energy if the product of E_v and the relaxation time t_v of the virtually excited state becomes

$$E_v t_v = 2\hbar. \quad (1)$$

Furthermore, the virtual energy is connected with virtually excited momenta via the dispersion relation $E(\mathbf{p})$ of the treated system. For the virtual momentum $\mathbf{p}_v$ an expression similar to (1) may be formulated

$$\mathbf{p}_v \mathbf{x}_v = 2\hbar, \quad (2)$$

where $\mathbf{x}_v$ describes the mean path within which the virtual energy is lost. Therefore, the quantity $\mathbf{x}_v$ is called the virtual “relaxation path.” And, by means of performing virtual relaxation processes, in which the energies E_v and the momenta $\mathbf{p}_v$ are lost and exchanged, particles are able to interact with each other.

III. INTERPRETATION

With the introduction of a second excitation energy an additional point must be considered when treating a physical problem. This is achieved by checking if condition (1) is fulfilled. If this is the case, the situation changes from the picture developed so far and virtual excitation processes must be taken into account. The expression “virtual” is used for the new quantities since processes resulting from the postulate are seen to be of the same kind as virtual processes which have already been introduced in quantum theories. However, the approach within this work treats them from a new perspective. And, both virtual and thermal processes are treated in a related way. Thus, with the equipartition relation,

E_v is identified with a virtual temperature T_v too. The introduction of an additional excitation energy was motivated because it makes an energy source available which can provide the energy necessary for the creation of virtual particles which are used to mediate an interaction. The examinations done showed that the particles that exhibit a virtual energy were fermions. Whereas the ones that are created during a relaxation process, in which their creation is responsible for the decrease of E_v , were bosons. The last-mentioned particles are called virtual particles. If another kind of particle can be considered for the appearing particles remains open.

Since (virtual) relaxation processes play a key role in my concept, the in this context appearing quantities regarding time and space are interpreted next. Regarding time, there exists one such quantity. This is the relaxation time t_v which describes the mean period of time within which a particle loses its virtual excitation energy E_v due to virtual scattering processes. It is noted that this relaxation time is, in general, not equal to the period of time within which the virtual energy is most likely lost. Regarding space, there exist two quantities between which has to be distinguished. First, there is the relaxation path $\mathbf{x}_v$. It describes the mean path traveled by a particle in the direction of the virtual momentum $\mathbf{p}_v$ during a relaxation process. Second, there is the quantity $\mathbf{l}_v$ which is called the virtual “relaxation length” and which is introduced quantitatively in Subsec. IV A. It describes the mean total path traveled by a particle during a relaxation process. Two quantities are needed because a particle may exhibit a net momentum $\mathbf{p}_g$ with a finite part perpendicular to $\mathbf{p}_v$ which leads to a movement as well. In Fig. 2 they are depicted for an exemplary case. Now, the five quantities E_v , t_v , $\mathbf{p}_v$,

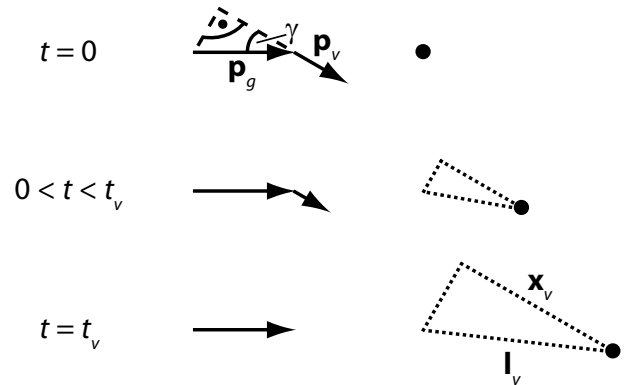


FIG. 2: Time-dependent momentum and path traveled by a particle during a relaxation process in which the energy E_v is lost within the period of time t_v . The total traveled path $\mathbf{l}_v$ constitutes of the part $\mathbf{x}_v$ which results from the virtual momentum $\mathbf{p}_v$ and the net momentum parallel to $\mathbf{p}_v$ plus the part which results from the net momentum perpendicular to $\mathbf{p}_v$. The angle between $\mathbf{p}_g$ and $\mathbf{p}_v$ is called γ .

$\mathbf{x}_v$, and $\mathbf{l}_v$, which build the cornerstones of the new concept, are introduced. Furthermore, I formulate an ex-

pression for the excitation-energy-dependent relaxation time which shall contain the decisive features of a relaxation process. There are seen to be three of them. First, there is the scattering frequency ν_s which describes the frequency of the scattering processes (i.e., the frequency of the creation of particles). Second, there is the mean number of particles $\bar{N}$ created at each scattering process. Third, there is the mean energy $\bar{E}$ of a created particle. If all features are taken into account I get

$$t(E) = \frac{1}{\nu_s} \frac{1}{\bar{N}\bar{E}} E, \quad (3)$$

because the period of time between two subsequent scattering processes equals ν_s^{-1} and at each the amount $\bar{N}\bar{E}$ of the excitation energy E is lost.

Next, the question that needs to be answered to find out if a particle exhibits a virtual energy is formulated again: In a given configuration, is it possible that a particle exhibits the energy E_v so that it would take the mean period of time $t_v = 2\hbar/E_v$ to lose it? To answer the question one needs to know a second relation which combines E_v and t_v . This relation has to be found by examining the given configuration. Furthermore, the magnitude of E_v is expected to depend on the present scattering mechanisms. To handle the behavior, they are divided into two forms. To the first virtual scattering mechanisms, which are responsible for virtual scattering processes and thereby for the loss of E_v , belong. To the second real scattering mechanisms, which shall include all scattering mechanisms except the virtual ones, belong. In analogy to the virtual, the real scattering mechanisms are characterized by a real relaxation time t_r . Further it is assumed that a particle can only experience virtual and/or real scattering processes and not any kind of “mixture” of both, e.g., the creation of a particle due to the virtual as well as the thermal energy. A change of the magnitude of E_v can now be expected because a particle experiences real optionally to virtual scattering processes. This influence is illustrated in Fig. 3 and is discussed in the following. The model I introduce to describe the influence works with the assumption that a particle sees points in space where it is possible to create virtual particles through a virtual scattering process. The assumption is inspired by the situation of a free electron in a solid. There, the electron is moving through a lattice of ions and every ion can be excited by means of the creation of a particle, e.g., the creation of a phonon. Thus, every ion stands for such a point. Furthermore, while moving through a medium, a particle experiences virtual and/or real scattering processes with properties given by the present scattering mechanisms. In case of Fig. 3(a) only virtual scattering mechanisms are present. In case of Figs. 3(b) and 3(c) real scattering mechanisms are present additionally. This leads to the situation that, with a finite probability, a particle experiences a real instead of a virtual scattering process which makes it more difficult to lose E_v since the loss

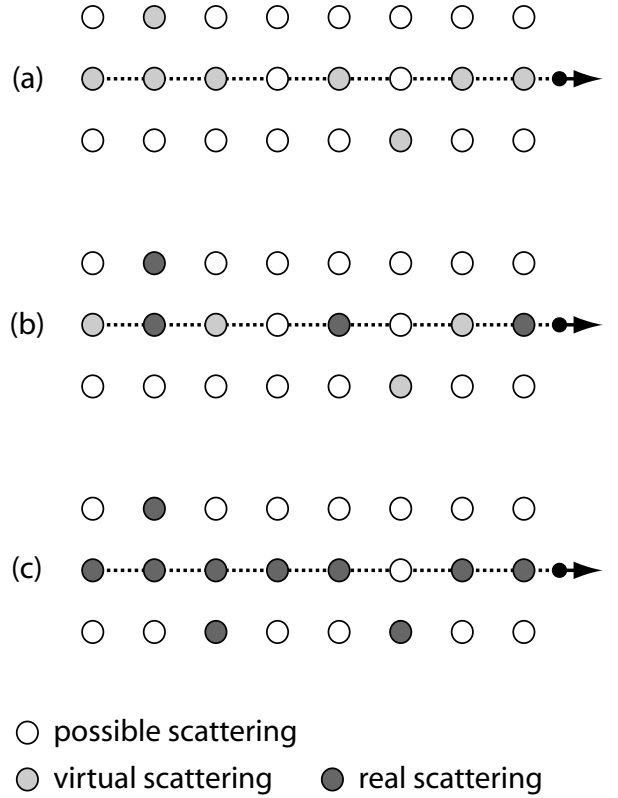


FIG. 3: Influence of real scattering mechanisms on a virtual relaxation process. For movement (a) only virtual scattering mechanisms are present. For movements (b) and (c) real ones are present additionally, differently strong.

arises from virtual scattering processes. Hence, the relaxation time increases and, in order to still satisfy condition (1), E_v has to decrease. This kind of decrease is possible in Fig. 3(b), whereas in Fig. 3(c) the real dominate the virtual scattering mechanisms so that they do not take place and the virtual energy vanishes. At last, I present the formalism with which the second relation which combines E_v and t_v is derived. To begin with, the real scattering mechanisms are neglected. Then, only virtual ones must be taken into account. In the next step, the excitation-energy-dependent relaxation time $t(E)$ of the system⁷ is inserted into (1) by replacing t by t_v and E by E_v . If function $t(E)$ allows the product Et to become $2\hbar$, one gets the two quantities E_0 and t_0 and a virtual energy may emerge. Furthermore, the obtained results serve as a basis for describing real systems with real scattering mechanisms. By examining the interplay between the virtual and the real scattering mechanisms it is then possible to determine the change of E_v from E_0 . The consequences of the interplay are shown in Fig. 4, which can be compared with Fig. 3.

After determining E_v , it is possible to determine the virtual momenta. For that, the virtual energy is expressed as

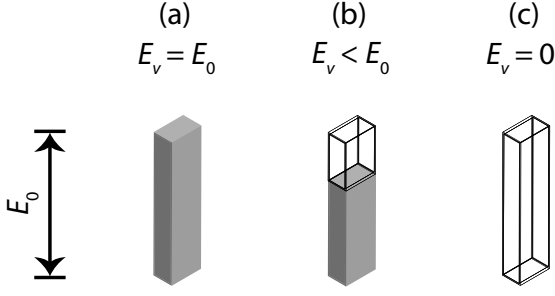


FIG. 4: Influence of real scattering mechanisms on the magnitude of E_v . Starting from neglecting them (a), their presence is increased (b), and, finally, their presence is increased so much that they dominate the virtual scattering mechanisms (c).

$$E_v = E_e - E_g, \quad (4)$$

where E_e is the energy of the virtually excited state and E_g of the ground state. An analog expression is formulated for the virtual momentum

$$\mathbf{p}_v = \mathbf{p}_e - \mathbf{p}_g, \quad (5)$$

where $\mathbf{p}_e$ is the momentum of the virtually excited state and $\mathbf{p}_g$ of the ground state. By combining the dispersion relation $E(\mathbf{p})$ with (4) and (5) one obtains the virtual momenta $\mathbf{p}_v$ which lead to an excitation energy E_v . From $\mathbf{p}_v$ again, the relaxation path $\mathbf{x}_v$ can be determined with relation (2). This can be done because it is assumed that the part of $\mathbf{x}_v$ perpendicular to $\mathbf{p}_v$ vanishes, i.e., $\mathbf{x}_v \parallel \mathbf{p}_v$. The assumption rests on the interpretation of $\mathbf{x}_v$ being the mean path traveled in the direction of $\mathbf{p}_v$. Hence, relation (2) can be put to

$$p_v x_v = 2\hbar, \quad (6)$$

when one is interested in the relationship between the absolute values of $\mathbf{p}_v$ and $\mathbf{x}_v$.

Now, the phenomenon how a particle maintains a constant virtual energy is treated. At first, some general remarks are made. In physical systems particles tend to occupy states with a minimum energy. To get into the energetically lowest lying state, i.e., the ground state, a particle gives off its excitation energy to its surrounding medium. Furthermore, the surrounding medium exhibits in general also mechanisms responsible for a gain of energy. In an equilibrium both mechanisms cancel each other out and a particle maintains a constant excitation energy. Since a particle in such a situation never loses its excitation energy, relaxation processes are treated as hypothetical ones by neglecting the gain of energy. Next, we return to the maintaining of the virtual energy. In case of the thermal energy of an electron in a solid, the

gain arises from the annihilation of thermally excited particles. The corresponding mechanism for the virtual energy E_v is assumed to work as follows: A particle “conjures up” virtual particles and uses them to maintain E_v . By the expression “conjure up” I mean that virtual particles are created out of nothing and the state of the particle remains unchanged. To prevent the loss of virtual energy, which is assumed to arise solely from the creation of virtual particles, each time a virtual particle is created a conjured up virtual particle is also annihilated. The annihilated possess equivalent properties to the created particles and compensate the loss. Furthermore, the created particles vanish. In order that there is enough energy available, a particle conjures up virtual particles with the total energy E_v . These are chosen so that they compensate for the created particles in the hypothetical relaxation process. This leads to the situation that the particle is constantly surrounded by virtual particles which would be created during a relaxation process. In Fig. 5 these considerations are depicted.

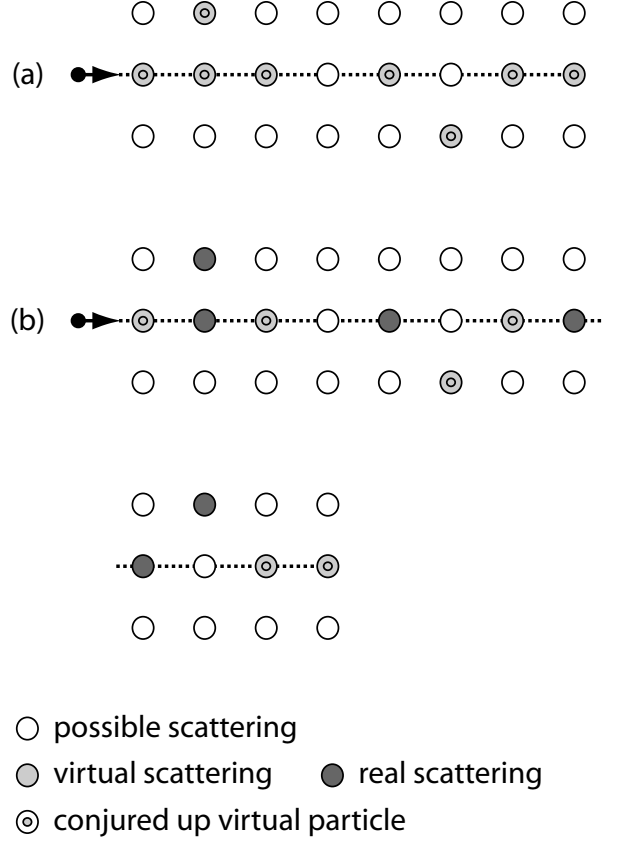


FIG. 5: Conjuring up of virtual particles to maintain E_v . For relaxation process (a), E_v would be lost through the creation of 8 virtual particles. For relaxation process (b), E_v would be lost through the creation of 6 virtual particles. Virtual particles are conjured up according to the relaxation process.

Up to now, the interpretation was carried out in a single-particle picture in which a single particle is moving in a potential which determines its states and how tran-

TABLE I: Symbols to describe the quantities related to a positive and a negative virtual energy.

	Excitation energy	Relaxation time	Momentum	Relaxation path
+	E_v	t_v	$\mathbf{p}_v$	$\mathbf{x}_v$
−	Δ_v	τ_v	$\boldsymbol{\pi}_v$	$\boldsymbol{\xi}_v$
	Relaxation length	Temperature	Velocity	
+	$\mathbf{l}_v$	T_v	$\mathbf{v}_v$	
−	$\boldsymbol{\lambda}_v$	Υ_v	$\boldsymbol{\nu}_v$	

sitions between them take place. By that the dispersion relation and the excitation-energy-dependent relaxation time are determined and one can describe the virtual processes. However, as soon as virtual processes of different particles influence each other, the single-particle must be extended to a many-particle picture. There were found two scenarios in which the influence becomes apparent. The first in which additional virtual particles influence the relaxation time is only noticeable in the solid-state configuration of Subsec. IV C. The second manifests itself in the emergence of a new interaction and is used to explain the interaction that causes superconductivity. Whereas the first scenario is treated in Subsec. IV C, the final part of the interpretation is dedicated to the second scenario which is of great importance for the understanding of physics.

To begin with, I remind that so far virtual relaxation processes were treated hypothetically. Consequently, a particle remains in its virtually excited state. But in principle, every particle that exhibits a virtual energy is able to perform a virtual relaxation process. Actually, the event that a particle uses this ability and performs a virtual relaxation process is seen to be equivalent to the situation which is in the picture developed so far known as “a particle undergoes a change of the state of motion because of experiencing an interaction.” Thus, the picture developed within this work says that the consequences of a force acting on a particle can be described by relaxation processes. By that also the quantities t_v , $\mathbf{x}_v$, and $\mathbf{l}_v$ become useful in a concrete way. Furthermore, through a relaxation process, a particle gives off on average a part of its virtual energy in form of virtual particles in the surrounding medium. The part given off is identified with a negative virtual excitation energy Δ_v and quantities related to it are used to describe the movement. To distinguish between both forms of appearance, these quantities are represented by Greek letters. In Table I all quantities with their symbols are listed. As for the positive virtual processes, a negative virtually excited state is assigned to a particle which reduces the difference between both to the algebraic sign. Now, the quantities related to a negative virtual energy are interpreted. The negative virtual energy describes the energy given off on average through a performed virtual relaxation process and, analogously, the negative virtual

momentum describes the momentum given off on average through a performed virtual relaxation process. Furthermore, since the virtual temperature and the virtual velocity are connected with the virtual energy and the virtual momentum, the interpretation holds also for them. In addition, it implies that $\boldsymbol{\pi}_v \parallel \mathbf{p}_v$ and $\boldsymbol{\nu}_v \parallel \mathbf{v}_v$. The remaining quantities, namely, the relaxation time, the relaxation path, and the relaxation length are equivalent to the positive, $t_v = \tau_v$, $\mathbf{x}_v = \boldsymbol{\xi}_v$, and $\mathbf{l}_v = \boldsymbol{\lambda}_v$. In spite of the equivalence new symbols are introduced to be able to tell which form of appearance is used in the description. At last, I derive the relationship between the quantities which are not equivalent, $E_v \neq \Delta_v$, $\mathbf{p}_v \neq \boldsymbol{\pi}_v$, $T_v \neq \Upsilon_v$, and $\mathbf{v}_v \neq \boldsymbol{\nu}_v$, for a free particle with the dispersion relation $E = \mathbf{p}^2/2m$, where m is its mass. The energy and the momentum given off are obtained by equating two expressions for the relaxation path x_v . To begin with, the connection between E_v and $\mathbf{p}_v$ is formulated according to (4) and (5)

$$\begin{aligned} E_v &= \frac{1}{2m}(\mathbf{p}_g + \mathbf{p}_v)^2 - \frac{1}{2m}\mathbf{p}_g^2 \\ &= \frac{1}{2m}(2p_g p_v \cos \gamma + p_v^2), \end{aligned} \quad (7)$$

where γ is the angle between $\mathbf{p}_g$ and $\mathbf{p}_v$. Next, the first expression is written, with the aid of (6), as

$$x_v = \frac{2\hbar}{p_v}. \quad (8)$$

The second expression is written as

$$x_v = \bar{v}' t_v, \quad (9)$$

where $\bar{v}'$ describes the mean velocity during a relaxation process in the direction of $\mathbf{p}_v$. Then, by equating (8) and (9) and using (1) and (7), one obtains

$$\bar{v}' = v_g \cos \gamma + \frac{1}{2}v_v, \quad (10)$$

when one connects the velocity $\mathbf{v}$ with the momentum $\mathbf{p}$ via $\mathbf{v} = \mathbf{p}/m$. Since the term “ $v_g \cos \gamma$ ” corresponds to an offset part of the net velocity in the direction of $\mathbf{p}_v$, the part of the virtual velocity given off on average becomes $v_v/2$. Furthermore, since the sum of the positive and the negative virtual velocity must equal $v_v/2$ and $\boldsymbol{\nu}_v \parallel \mathbf{v}_v$, one gets

$$\boldsymbol{\nu}_v = \frac{1}{2}\mathbf{v}_v, \quad (11)$$

$$\boldsymbol{\pi}_v = \frac{1}{2}\mathbf{p}_v. \quad (12)$$

Next, an expression analog to (7) is formulated for the negative virtual energy

$$\Delta_v = \frac{1}{2m}(2p_g\pi_v \cos \gamma + \pi_v^2), \quad (13)$$

where γ is the angle between $\mathbf{p}_g$ and π_v which equals the one between $\mathbf{p}_g$ and $\mathbf{p}_v$. In Fig. 6 the results are illustrated. And finally, with the equipartition relation

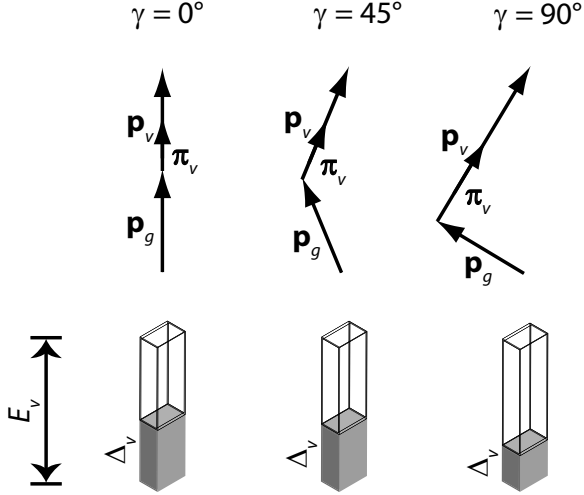


FIG. 6: Three configurations of $\mathbf{p}_v$ for the identical value E_v and the quantities Δ_v and π_v resulting from a performed relaxation process.

for three translational degrees of freedom, Δ_v is identified with a negative virtual temperature

$$\Delta_v = \frac{3}{2}k_B\Upsilon_v. \quad (14)$$

The concluding paragraph deals with the interaction predicted by the postulate.

The question that needs to be answered is how the change of the state of motion can be described from the perspective of relaxation processes. Since this perspective is new, a new set of rules needs to be introduced to make use of it. Therefore, I introduce one containing three rules after which interactions via virtual relaxation processes are treated. The three rules are:

RULE I: If an interaction takes place, then in the form that a particle first performs a virtual relaxation process and, directly thereafter, performs a process inverse to a virtual relaxation process performed by another particle. Within the inverse relaxation process virtual particles are annihilated in inverse order as they were created. This means that the energy and the momentum lost by one particle is gained by another particle and the other way around. Throughout the exchange no

conservation laws are violated which means that every created virtual particle must possess the right properties so that it can be annihilated.

RULE II: Particles interact with each other if under all virtual momenta every particle exhibits there are two parallel to each other. Additionally, the two momenta must lie in line when they point through the endpoints of every particle after performing the relaxation process. If these conditions are fulfilled, particles perform virtual relaxation processes by losing the corresponding momenta.

RULE III: The virtual energy is lost through the creation of a virtual particle and gained through the annihilation of a virtual particle at a point which exhibits no conjured up virtual particle. Thus, the magnitude of Δ_v is determined by the probability of finding a free point. After the relaxation processes virtual particles are relocated so that they are annihilated within the inverse relaxation processes.

In Figs. 7, 8, and 9 all rules are illustrated.

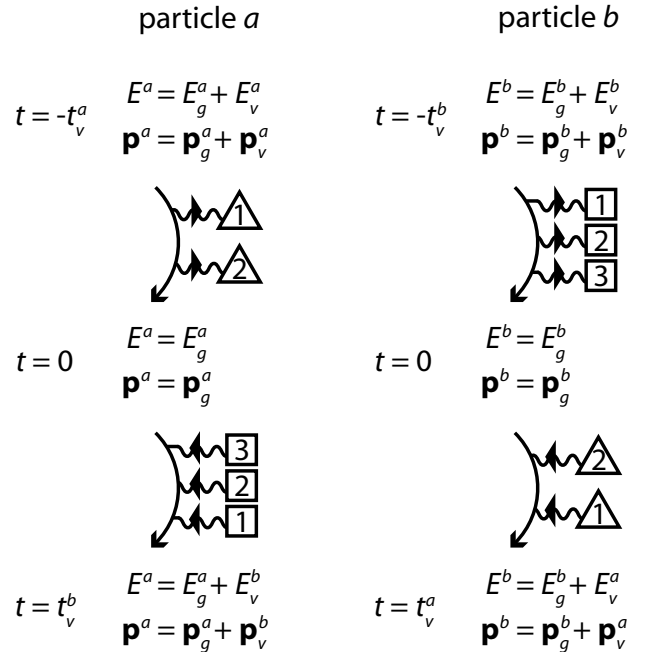


FIG. 7: Interaction between two particles via virtual relaxation processes in which the virtual energies are lost through the creation of 2 and 3 virtual particles (Rule I).

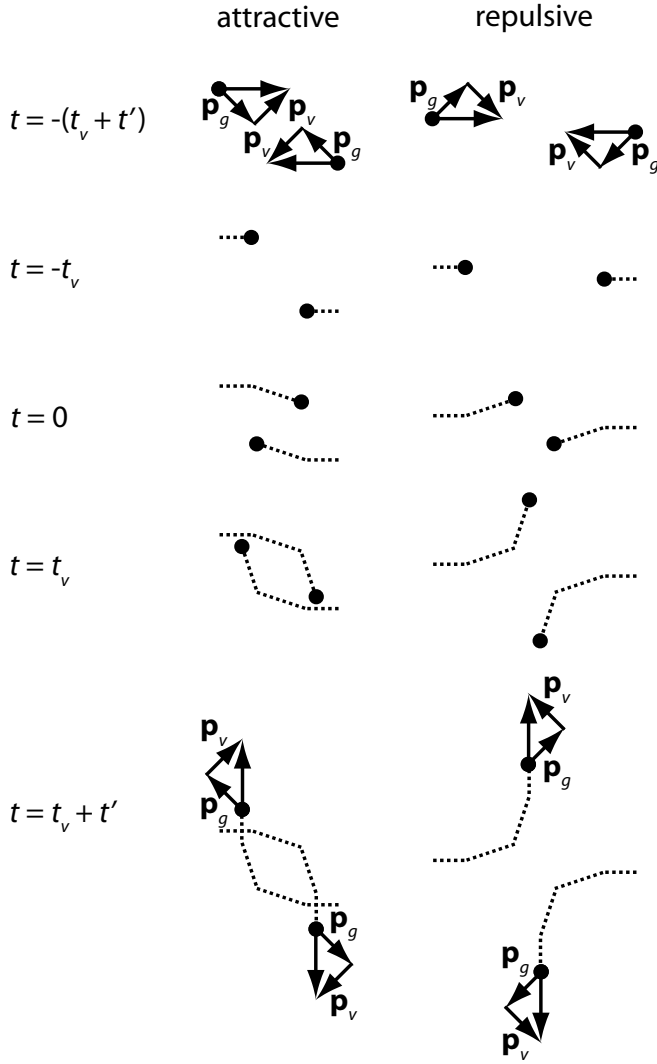


FIG. 8: Time-dependent traveled paths of two interacting particles (Rule II). It is chosen that $t_v^a = t_v^b = t_v$, $p_v^a = p_v^b = p_v$, $p_g^a = p_g^b = p_g$, and $\gamma = 90^\circ$. Depending on in which direction p_v is lost, one can construct an attractive and a repulsive interaction.

IV. APPLICATION TO SUPERCONDUCTIVITY (PART ONE)

In this and the next section the postulate is applied to solid-state physics and thereby to superconductivity. In the first part virtual energies E_v in two solid-state configurations are determined. The results are the starting point for further investigations in Sec. V. Before focusing on the two materials, the relaxation length is introduced quantitatively.

A. Relaxation length

The virtual relaxation length l_v describes the mean total path traveled by a particle during a relaxation process

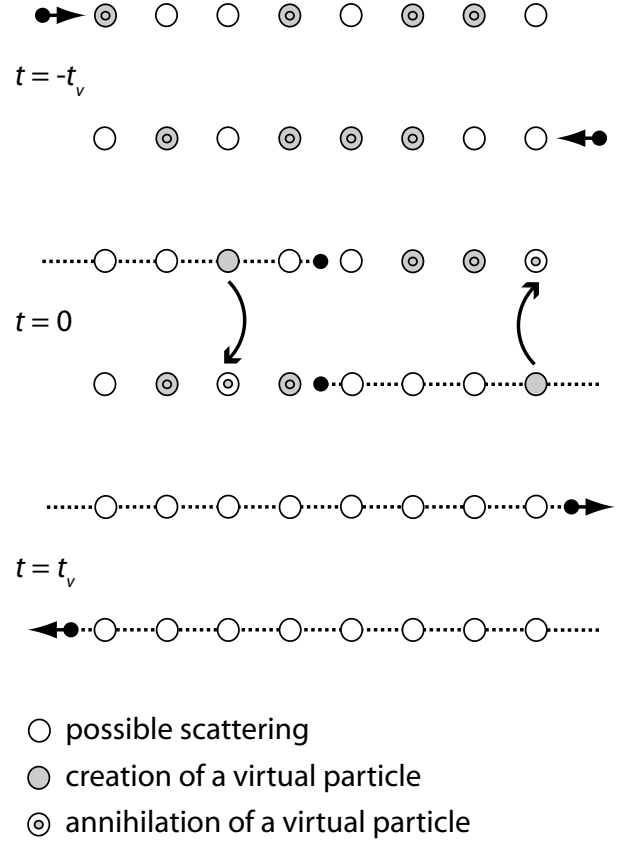


FIG. 9: Exchange of virtual particles (Rule III). A virtual particle can only be exchanged if there are two points which exhibit no conjoined up virtual particle.

and thus, if $\bar{\mathbf{v}}$ is the mean velocity during the relaxation process and (1) is used, one obtains

$$\mathbf{l}_v = \bar{\mathbf{v}} t_v = \bar{\mathbf{v}} \frac{2\hbar}{E_v}. \quad (15)$$

Furthermore, with the aid of (10), one gets

$$\bar{\mathbf{v}} = \mathbf{v}_g + \frac{1}{2} \mathbf{v}_v. \quad (16)$$

If $v_g = 0$, l_v and x_v must equal each other. This can be checked by inserting (16) into (15) and using $E = m\mathbf{v}^2/2$,

$$l_v = \frac{2\hbar}{mv_v} = \frac{2\hbar}{p_v} = x_v. \quad (17)$$

B. Normal metals

The first configuration the postulate is applied to are the normal metals. Within this article materials are called “normal” metals if their temperature-dependent relaxation time for electrons can be described by the Bloch-Grüneisen law

$$t(T) = A \left(\frac{\Theta}{T} \right)^5 J^{-1} \left(\frac{\Theta}{T} \right), \quad (18)$$

where A is a material-dependent constant, Θ the Debye temperature, and $J(\Theta/T)$ the integral

$$J \left(\frac{\Theta}{T} \right) = \int_0^{\frac{\Theta}{T}} \frac{x^5}{(e^x - 1)(1 - e^{-x})} dx. \quad (19)$$

In the following, it is assumed that the temperatures with which is dealt are at maximum of the order of 50 K and that $50 \text{ K} \ll \Theta$.⁸ Then, integral (19) becomes

$$\lim_{1 \ll \frac{\Theta}{T}} \int_0^{\frac{\Theta}{T}} \frac{x^5}{(e^x - 1)(1 - e^{-x})} dx = 124, \quad (20)$$

and (18) can be reduced to

$$t(T) = \frac{A}{124} \left(\frac{\Theta}{T} \right)^5. \quad (21)$$

Function (21) was derived for electrons relaxing due to scattering on phonons. The T^{-5} -dependence is characteristic for electron-phonon scattering, whereas a T^{-2} -dependence is found for electron-electron scattering and, in addition, s-d electron scattering may contribute to the scattering.⁹ Furthermore, since electrons in metals can be described by a free-electron-gas model, the equipartition relation for three translational degrees of freedom

$$E = \frac{3}{2} k_B T \quad (22)$$

is used to identify E with T . Now, it is possible to convert (21) into $t(E)$. Equating (1) and (21) and using (22) yields then

$$T_0 = \left(\frac{k_B A \Theta^5}{165 \hbar} \right)^{\frac{1}{4}}, \quad (23)$$

$$E_0 = \frac{3}{2} k_B \left(\frac{k_B A \Theta^5}{165 \hbar} \right)^{\frac{1}{4}}. \quad (24)$$

To calculate T_0 , the quantity $A \Theta^5$ is determined by means of the electrical resistivity given by the Drude formula

$$\rho = \frac{m}{n e^2 t}, \quad (25)$$

where m is the free electron mass and n the density of free electrons. Combining (21) with (25) yields

TABLE II: Experimental values of the electrical resistivity ρ at $T = 77 \text{ K}$ and the density of free electrons n . The quantity $A \Theta^5$ is determined by (26) and T_0 by (23).

Normal metal	Experiment ^a		Theory	
	ρ ($10^{-8} \Omega \text{m}$)	n (10^{28} m^{-3})	$A \Theta^5$ (10^{-3} sK^5)	T_0 (K)
Cd	1.6	9.27	8.03	50
Zn	1.1	13.2	8.2	51
Ga	2.75	15.4	2.81	39
Al	0.3	18.1	21.93	65
Tl	3.7	10.5	3.07	40
In	1.8	11.5	5.75	46
Sn	2.1	14.8	3.83	42
Hg	5.8	8.65	2.37	37
Pb	4.7	13.2	1.92	35

^aN. W. Ashcroft and D. N. Mermin, *Festkörperphysik* (Oldenbourg, München, 2007).

$$A \Theta^5 = \frac{m}{\rho n e^2} 124 T^5. \quad (26)$$

In Table II experimental values of ρ at $T = 77 \text{ K}$ and n for different normal metals are listed plus $A \Theta^5$ and T_0 are calculated.

Next, the interplay between virtual and real scattering mechanisms is examined. I restrict the examinations to real scattering mechanisms resulting from thermal scattering mechanisms, i.e., with the ones resulting from lattice imperfections is not dealt and about their influence can only be said that it is expected to have the qualitative appearance discussed in the interpretation. To obtain the quantitative influence of thermal scattering mechanisms, I look at how E_v changes from E_0 . Since their increasing presence decreases E_v , the following equation is formulated

$$E_v = E_0 - E_t, \quad (27)$$

where E_t is the part of E_0 which cannot be lost because, with a finite probability, an electron experiences thermal instead of virtual scattering processes. By making the assumption that the scattering probability is proportional to the inverse relaxation time t^{-1} , one can write

$$E_t = E_0 \frac{P_t}{P_0} = E_0 \frac{t_0}{t_t}, \quad (28)$$

where P_t is the probability for thermal scattering, P_0 for virtual scattering, and t_t is the thermal relaxation time. Then, by combining (27) and (28) with (21), one gets

$$E_v = E_0 \left[1 - \left(\frac{T}{T_0} \right)^5 \right]. \quad (29)$$

Thus, the virtual energy decreases with increasing T and vanishes for $T > T_0$.

Finally, I show that a form of Matthiessen's rule holds also for the interplay between virtual and thermal scattering mechanisms. This can be seen by inserting (28) into (27) and using (1)

$$\frac{1}{t_v} = \frac{1}{t_0} - \frac{1}{t_t}. \quad (30)$$

C. Antiferromagnetic lattice

The second configuration the postulate is applied to is an antiferromagnetic lattice. I restrict the application to a two-dimensional quadratic lattice as it can be found in a cuprate superconductor in form of the CuO_2 plane. However, by using a similar approach, it should also be possible to apply it to other configurations. To begin with, a summary of the properties and decisive mechanisms of the CuO_2 plane, as they look like from my point of view, is made, which is a proposal how the CuO_2 plane can be treated which is still an unsettled question.

In the undoped insulating CuO_2 plane every magnetic moment, connected with a total spin $S \neq 0$, can be assigned to a copper ion with the electronic configuration $[\text{Ar}]3d^9$ and $S = 1/2$. The oxygen ions possess $S = 0$ and are responsible for the antiferromagnetic coupling. Adding an electron to (electron-doping) and removing one from the CuO_2 plane (hole-doping) leads to $[\text{Ar}]3d^{10}$ and $[\text{Ar}]3d^8$. And, from Hund's rules follows that the total spin becomes $S = 0$ and $S = 1$, see Fig. 10. Fur-

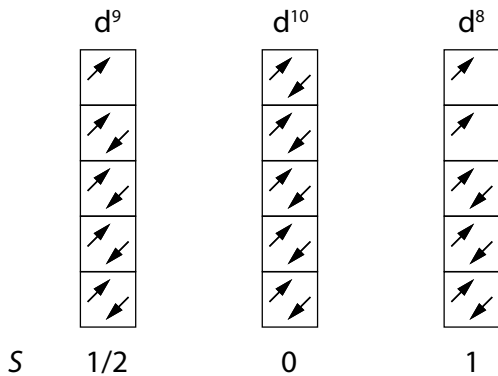


FIG. 10: Copper ions at different states of doping.

thermore, there are two mechanisms which determine the properties of the lattice. First, a strong Coulomb interaction $[O(1 \text{ eV})]$ leads to a localization of the d-electrons at the lattice points of the copper ions. Second, between nearest-neighbor localized electrons acts an antiferromagnetic coupling $[O(100 \text{ meV})]$ leading to the arrangement shown in Fig. 11.

Now, I continue with the virtual processes. To begin with, the function $t(E)$ has to be found. Because of the lack of such an established function, a new model is

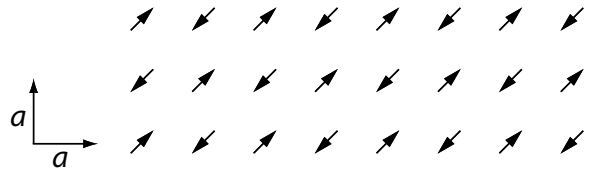


FIG. 11: Antiferromagnetic arrangement of the magnetic moments, where a is the lattice constant of the quadratic lattice.

introduced in which the movement of a charge carrier is described by a series of hops. It is assumed that the hops a charge carrier is able to perform consist of hops to one of the eight nearest-neighbor lattice points of the copper ions and that the path between two lattice points is taken so that a minimum number of hops is needed.¹⁰ The eight hops are divided into two groups. To the first hops in the direction $\alpha = j \times 90^\circ$, where $j = 0, 1, 2, 3$, belong (α_{90°). To the second hops in the direction $\alpha = k \times 45^\circ$, where $k = 1, 3, 5, 7$, belong (α_{45°). In Fig. 12 the hops with the angle α are shown. The question that needs to be

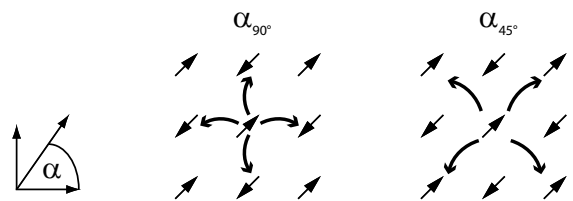


FIG. 12: The eight hops a charge carrier is able to perform. From them the movement is constructed.

answered now is how much energy each hop costs. In the undoped configuration, where the Coulomb interaction dominates, a hop requires an excitation energy of the order of 1 eV. Because of the largeness of the value compared to typical excitation energies (e.g., the thermal excitation energy at room temperature is of the order of 50 meV), most electrons are not able to hop and become localized, which manifests itself as an insulating behavior and no virtual processes occur. However, the situation changes as soon as the CuO_2 plane is doped and some lattice points possess a charge $\pm e$ compared to others. Whereas in the undoped a hop of an electron means that the charge of one lattice point becomes $2e$ larger than of a second, which causes the large energetic costs, in the doped configuration a hop of a charge carrier means only that the charge states of two lattice points are exchanged as it is illustrated in Fig. 13. Therefore, a hop of a charge carrier costs no energy due to the Coulomb interaction and the mechanism responsible for the loss remains the antiferromagnetic coupling. How it looks like is discussed next. For a better understanding in Figs. 14 and 15 the considerations to come are illustrated. For electrons, as for holes, it is assumed that a hop is performed into an identical state of the 3d subshell as from which is hopped (i.e., every lattice point possesses one state into which can be hopped, where the state can be occupied by two

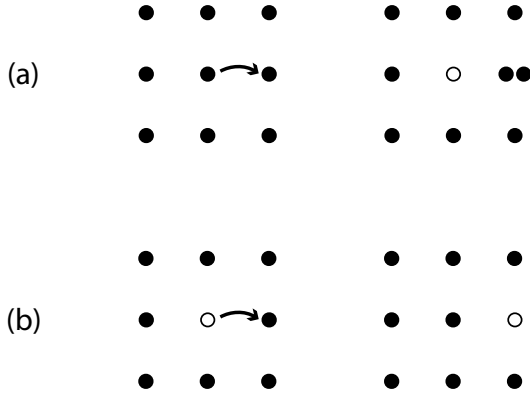


FIG. 13: In the undoped configuration (a) a hop leads to an energetically more expensive redistribution of the electrons, whereas in the doped configuration (b) it leads to an energetically equivalent redistribution.

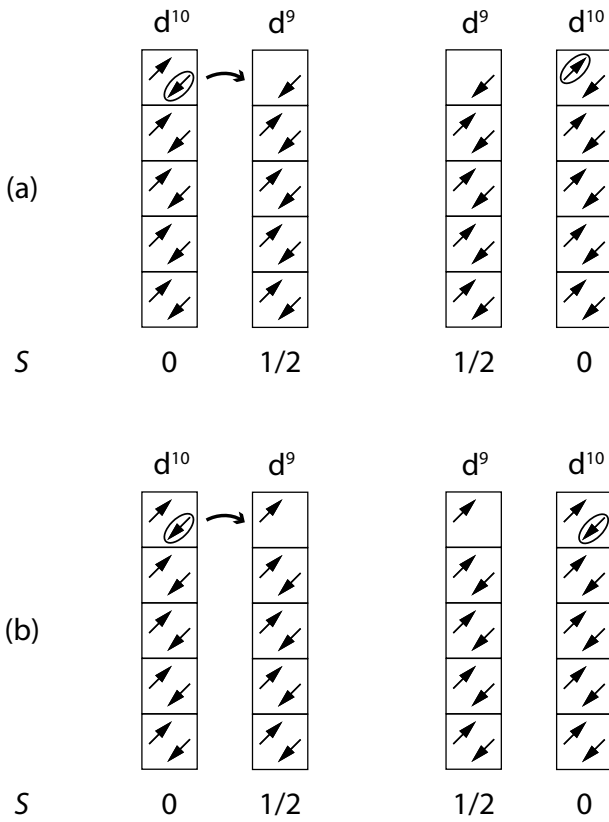


FIG. 14: A hop of an electron in the direction α_{90° (a) and α_{45° (b). The electron is marked with an ellipse.

electrons of opposite spin). First, hops of electrons in the direction α_{90° are examined [see Fig. 14(a)]. Since the state into which the electron is allowed to hop is occupied, it has to flip its spin. To conserve the total spin, another electron has to flip its spin too. This is achieved by flipping the spin of an electron of the lattice point from which is hopped. Since the energetically highest lying state must be the one which is only occupied by one

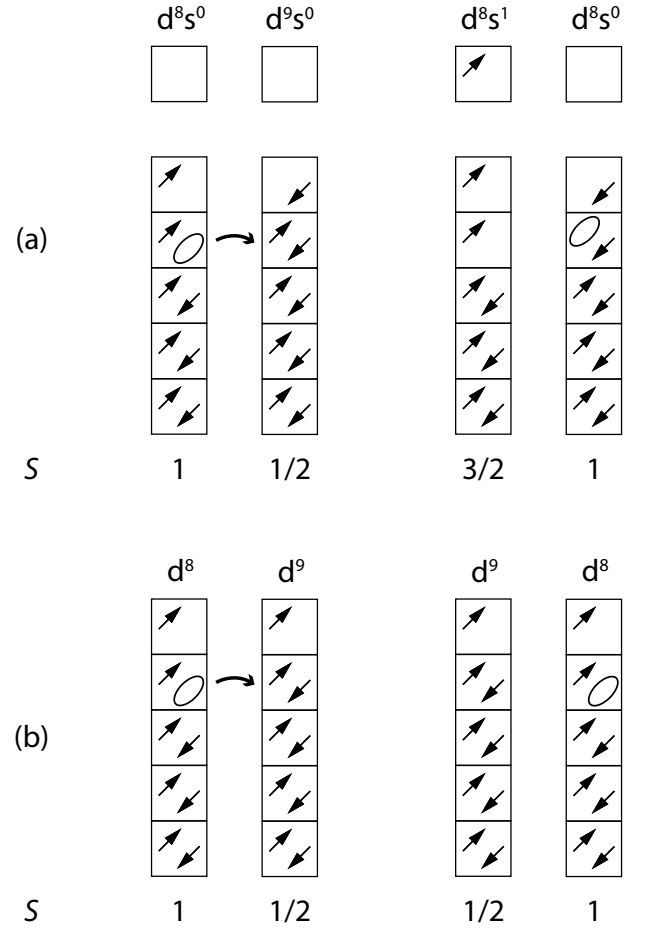


FIG. 15: A hop of a hole in the direction α_{90° (a) and α_{45° (b). The hole is marked with an ellipse.

electron, its spin-flip costs the least energy and is taken. However, the spin-flip costs energy because now one magnetic moment is aligned ferromagnetically in contrast to the favored antiferromagnetic alignment. These costs are called $\Xi_{\frac{1}{2}}$. Second, hops of electrons in the direction α_{45° are examined [see Fig. 14(b)]. Since there exists an unoccupied state into which the electron may hop, the hop costs no energy. Third, hops of holes in the direction α_{90° are examined [see Fig. 15(a)]. In this case, there exists a completely unoccupied state into which the hole may hop. Since the hop which leads to $S = 0$ is expected to be not possible due to the large costs of creating such a configuration, it is forbidden. Thus, the hole hops so that the electronic configuration remains the same with $S = 1$. This kind of hop requires that the total spin of the lattice point from which is hopped becomes $S = 3/2$, which is assigned to $[\text{Ar}]3d^84s^1$. The costs of increasing the spin are called $\Xi_{\frac{3}{2}}$. Fourth, hops of holes in the direction α_{45° are examined [see Fig. 15(b)]. Since there exists an unoccupied state into which the hole may hop, the hop costs no energy. In Fig. 16 the movement based on the new model is illustrated. Before continuing with the derivation of $t(E)$, some remarks about the states

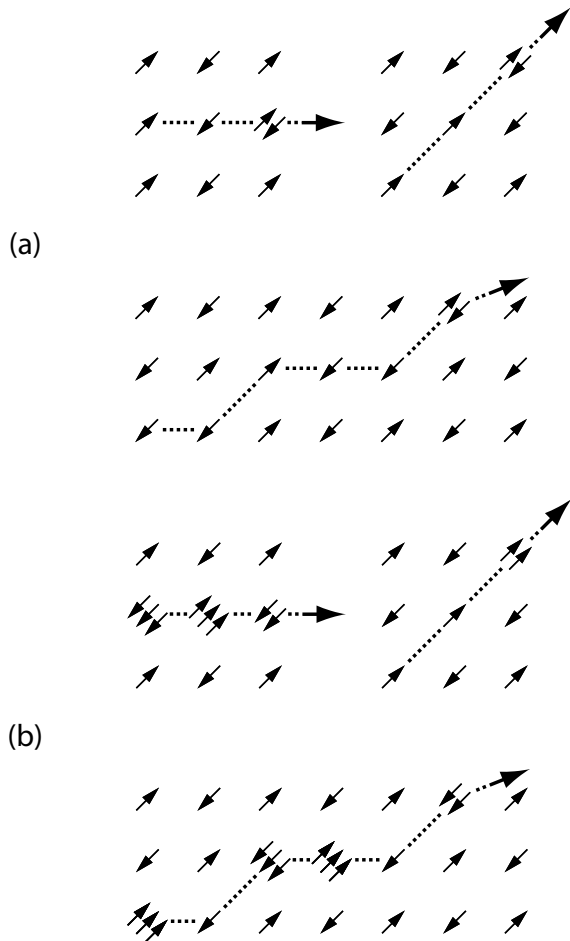


FIG. 16: Movement of an electron (a) and a hole (b) in the directions $\alpha = 0^\circ$, $\alpha = 45^\circ$, and $\alpha = 21.8^\circ$. The charge carrier is moving through a perfect antiferromagnetic lattice, where each hop in the direction α_{90° leaves behind an excited state.

of the electrons are made. Up to now, the treatment was carried out by using the states of an isolated copper ion. In the undoped configuration this seems reasonable because no hopping takes place which is seen to be connected with an overlap of wave functions and thereby with a modification from the states of an isolated copper ion. Because of this and $\Xi_{\frac{1}{2}}, \Xi_{\frac{3}{2}} \ll 1 \text{ eV}$, it is also expected that the quantities $\Xi_{\frac{1}{2}}$ and $\Xi_{\frac{3}{2}}$ are well-defined energies. However, a modification occurs as soon as electrons hop. Then, an overlap takes place which leads to that the well-defined energy levels turn into, effectively, a continuum of energy levels, i.e., an energy band. In the undoped configuration this case arises at excitation energies of the order of 1 eV and is thus not considered. In the doped configuration, though, a charge carrier is able to hop freely in the direction α_{45° and at excitation energies of the order of $\Xi_{\frac{1}{2}}, \Xi_{\frac{3}{2}}$ in the direction α_{90° . This is why electrons and holes are treated by an energy band.¹¹ Furthermore, I note that the energy barriers for hops should be reflected in the band structure in form of

an isotropic energy gap of the order of 1 eV for electrons in the undoped, and of an anisotropic one vanishing in the direction α_{45° and having its maximum value of the order of $\Xi_{\frac{1}{2}}, \Xi_{\frac{3}{2}}$ in the direction α_{90° for charge carriers in the doped configuration. Now, all preparations are made and we return to $t(E)$. At first, it is assumed that the change of the direction of propagation $\Delta\alpha$ during a relaxation process is small, $\Delta\alpha \ll 45^\circ$, and can be neglected. This is supported by the expression

$$\Delta\alpha = \arctan \frac{p_v}{p_g} = \arctan \frac{p_v}{p_F}, \quad (31)$$

which is obtained for $\gamma = 90^\circ$ (cf. Fig. 2) and by identifying p_g with the Fermi momentum p_F , and which becomes small because of the expected largeness of p_F . To begin with, the relaxation length $l(E)$ is determined in the direction α_{90° . Since each hop costs the energy Ξ_x , where $x = 1/2$ for electrons and $x = 3/2$ for holes, l is estimated with the argument that a charge carrier performs approximately E/Ξ_x hops until the excitation energy E is lost, where each hop moves it forward the distance of a lattice constant a , which yields

$$l_{90^\circ}(E) = \frac{a}{\Xi_x} E, \quad (32)$$

i.e., expression (3) is used with $t\nu_s = l_{90^\circ}/a$, $\bar{N} = 1$, and $\bar{E} = \Xi_x$. Next, (32) is extended to all directions of propagation by multiplying it by l'/l_{90° , where l' is the total traveled path. In Fig. 17 the corresponding geometric configuration is shown. As can be seen l' can

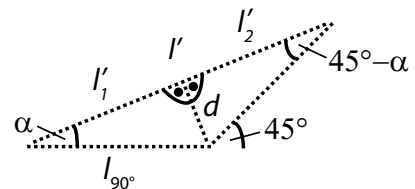


FIG. 17: Configuration from which the angle dependence of the relaxation length is obtained.

be written as

$$l' = l'_1 + l'_2 = \frac{d}{\tan \alpha} + \frac{d}{\tan(45^\circ - \alpha)}, \quad (33)$$

and d as

$$d = l_{90^\circ} \sin \alpha. \quad (34)$$

Then, by combining (33) and (34), one gets

$$\frac{l'}{l_{90^\circ}} = \cos \alpha + \frac{\sin \alpha}{\tan(45^\circ - \alpha)}. \quad (35)$$

And, multiplying (32) by (35) yields

$$l'(E) = \frac{a}{\Xi_x} \left[\cos \alpha + \frac{\sin \alpha}{\tan(45^\circ - \alpha)} \right] E, \quad (36)$$

where (36) holds for $0^\circ \leq \alpha \leq 45^\circ$ and can be extended to all angles because of symmetrical reasons. Now, $t'(E)$ is obtained from $l'(E)$ by $l' = \bar{v}t'$, where $\bar{v}$ is identified with the Fermi velocity v_F because of its expected largeness so that the second term in (16) can be neglected. And it is assumed that v_F is independent of α . Then, the excitation-energy-dependent relaxation time becomes

$$t'(E) = \frac{a}{\Xi_x v_F} \left[\cos \alpha + \frac{\sin \alpha}{\tan(45^\circ - \alpha)} \right] E, \quad (37)$$

and equating (1) and (37) yields

$$E'_0 = \sqrt{\frac{2\hbar\Xi_x v_F}{a}} \left[\cos \alpha + \frac{\sin \alpha}{\tan(45^\circ - \alpha)} \right]^{-\frac{1}{2}} \quad (38)$$

for the virtual energy a charge carrier takes on. Up to now, the derivation was carried out in a single-particle picture. This seems reasonable in respect to scattering between charge carriers because it is expected to be negligible, in analogy to electron-electron scattering in normal metals. However, one phenomenon is expected to influence relaxation processes, namely, the presence of conjured up virtual particles, where it is seen to be: Every conjured up virtual particle is located to one lattice point. And, if at one or both lattice points participating in a hop a virtual particle is located, the hop costs no energy. This is explained by the argument that there emerges an unoccupied state into which the charge carrier may hop without costs. Thus, a hop in the direction α_{90° costs only Ξ_x if at both lattice points no conjured up virtual particle is located, whereas a hop in the direction α_{45° is not effected since it already costs no energy. In Fig. 18 the thoughts are illustrated. In the next step, the number of virtual particles for a given number of charge carriers (for a given doping level) is determined. The interplay between conjuring up of virtual particles and their influence on relaxation processes is assumed to work as follows: Every charge carrier behaves in respect to conjuring up as it would as a single charge carrier. This means that, if X is the number of charge carriers and $\bar{N}$ of virtual particles a single charge carrier conjures up on average, the number of virtual particles becomes $\bar{N}X$. Now, charge carriers are moving through an anti-ferromagnetic lattice with $\bar{N}X$ excited lattice points. In the new configuration a hop in the direction α_{90° costs only Ξ_x if two lattice points are in a ground state. The probability of finding two is

$$P_{gg} = (1 - \bar{N}x)^2, \quad (39)$$

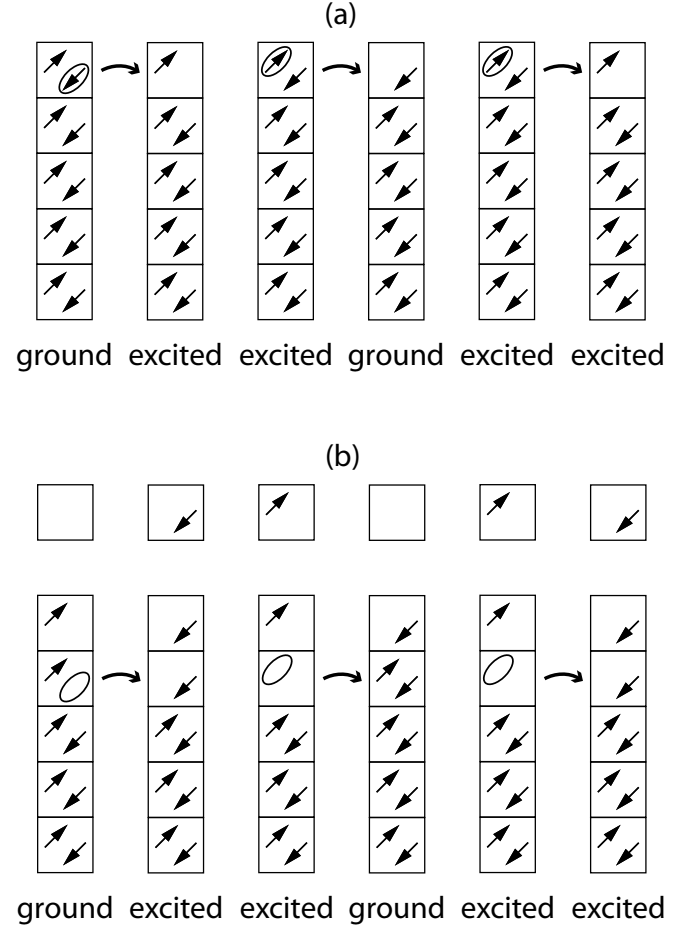


FIG. 18: If one or both lattice points participating in a hop in the direction α_{90° are excited, there emerges an unoccupied state into which an electron (a) and a hole (b) may hop without costs.

where x is the number of charge carriers per lattice point and $1 - \bar{N}x$ the probability of finding one ground state. In Appendix A the influence is examined and the result that (37) has to be multiplied by P_{gg}^{-1} is obtained. Next, $\bar{N}$ is calculated. The number of virtual particles a charge carrier conjures up is $N = E'_0/\Xi_x$, where E'_0 is given by (38). If all directions of propagation are treated equally, $\bar{N}$ may be obtained by averaging over α ,

$$\begin{aligned} \bar{N} &= \frac{4}{\pi} \int_0^{\frac{\pi}{4}} N d\alpha = \frac{4}{\pi} \int_0^{\frac{\pi}{4}} \frac{E'_0}{\Xi_x} d\alpha \\ &= \frac{4}{\pi} \int_0^{\frac{\pi}{4}} \left[\cos \alpha + \frac{\sin \alpha}{\tan(\frac{\pi}{4} - \alpha)} \right]^{-\frac{1}{2}} d\alpha \\ &\quad \times \sqrt{\frac{2\hbar v_F}{\Xi_x a}} \end{aligned} \quad (40)$$

$$\approx 0.69 \sqrt{\frac{2\hbar v_F}{\Xi_x a}}. \quad (41)$$

Therefore, the final excitation-energy-dependent relaxation time for charge carriers in the CuO_2 plane becomes

$$t(E) = \frac{a}{\Xi_x v_F} \left[\cos \alpha + \frac{\sin \alpha}{\tan(45^\circ - \alpha)} \right] \frac{E}{(1 - \bar{N}x)^2}, \quad (42)$$

where $t(E) = \infty$ for $1 - \bar{N}x < 0$ when all lattice points are excited. Function (42) was derived for charge carriers relaxing due to scattering on electronic excitations in form of bringing a valence electron of a copper ion in an energetically higher lying state, i.e., electron-electron and hole-electron scattering is responsible for relaxation processes. It is assumed that this scattering dominates as it was that in normal metals electron-phonon scattering does. Furthermore, the final virtual energy a charge carrier in the CuO₂ plane takes on becomes

$$E_0 = \sqrt{\frac{2\hbar\Xi_x v_F}{a}} \left[\cos \alpha + \frac{\sin \alpha}{\tan(45^\circ - \alpha)} \right]^{-\frac{1}{2}} \times (1 - \bar{N}x). \quad (43)$$

Temperature dependences are not examined. Thus, the presented views are a proposal how the movement of charge carriers can be described. However, of course, the qualitative behavior that E_v decreases with increasing T and vanishes for $T > T_0$ is expected.

V. APPLICATION TO SUPERCONDUCTIVITY (PART TWO)

The second part deals with all major observed phenomena connected with superconductivity and how they can be explained with the new concept, where it turns out that certain views on how to interpret the experiments are different from the present. In addition, it predicts phenomena neither observed nor predicted so far.

A. Superconducting interaction

The basis which determines the behavior of particles is the interaction between them. When examining charge carriers (electrons and holes) in a solid, their behavior was described with the electromagnetic interaction. Now, the situation changes because the postulate predicts that another interaction, which is called “superconducting,” emerges under a certain condition.¹² In the following, the behavior is examined by means of the concepts of Sec. III.

Firstly, it is assumed that only two electrons (holes) interact with each other by exchanging energies and momenta.¹³ After the exchange, though, both electrons may interact with other electrons. Secondly, it is assumed that an attractive interaction acts between them (cf. Fig. 8). Furthermore, electrons are treated with the ideal-Fermi-gas model, where the dispersion relation is given by $E = \mathbf{p}^2/2m$ in which m is the free electron

mass and all states with $E \leq E_F$ at $T = 0$ K are occupied. Since excitation energies are measured in respect to the Fermi energy E_F and the lowest lying unoccupied states lie at E_F , these states are identified with the ground states, $E_g = E_F$, $\mathbf{p}_g = \mathbf{p}_F$, and $\mathbf{v}_g = \mathbf{v}_F$. Then, Eq. (7) becomes

$$E_v = \frac{1}{2m} (2p_F p_v \cos \gamma + p_v^2). \quad (44)$$

Next, I work through the three rules on page 7. First, Rule I is examined. It is concerned with the conservation laws which require that the energy of the interacting electrons a and b before the exchange

$$\begin{aligned} E_v &= E_v^a + E_v^b \\ &= \frac{1}{2m} [2p_F^a p_v^a \cos \gamma^a + (p_v^a)^2] \\ &\quad + \frac{1}{2m} [2p_F^b p_v^b \cos \gamma^b + (p_v^b)^2] \end{aligned} \quad (45)$$

and after it

$$\begin{aligned} E'_v &= E_v^{a'} + E_v^{b'} \\ &= \frac{1}{2m} [2p_F^a p_v^b \cos \gamma^{a'} + (p_v^b)^2] \\ &\quad + \frac{1}{2m} [2p_F^b p_v^a \cos \gamma^{b'} + (p_v^a)^2] \end{aligned} \quad (46)$$

equal each other. Hence, because the momentum of the ground state is the same for all electrons, $p_F^a = p_F^b = p_F$, equating (45) and (46) yields

$$p_v^a \cos \gamma^a + p_v^b \cos \gamma^b = p_v^b \cos \gamma^{a'} + p_v^a \cos \gamma^{b'}. \quad (47)$$

Second, Rule II requires that $\mathbf{p}_v^a \parallel \mathbf{p}_v^b$ which implies that either $\gamma^{a'} = \gamma^a$ and $\gamma^{b'} = \gamma^b$ or $\gamma^{a'} = 180^\circ + \gamma^a$ and $\gamma^{b'} = 180^\circ + \gamma^b$, as one can make oneself clear from Fig. 2. The case that $\gamma^{a'} = \gamma^a$ and $\gamma^{b'} = \gamma^b$ is not considered because it is assumed that the interaction dominates for electrons whose virtual momenta $\mathbf{p}_v^a$ and $\mathbf{p}_v^b$ as well as whose momenta of the ground state $\mathbf{p}_F^a$ and $\mathbf{p}_F^b$ show in the opposite direction. Thus we have $\gamma^{a'} = 180^\circ + \gamma^a$ and $\gamma^{b'} = 180^\circ + \gamma^b$ which implies that $\cos \gamma^a = -\cos \gamma^{a'}$ and $\cos \gamma^b = -\cos \gamma^{b'}$, and (47) becomes

$$\begin{aligned} (p_v^a + p_v^b) \cos \gamma^a + (p_v^a + p_v^b) \cos \gamma^b &= 0 \\ \cos \gamma^a + \cos \gamma^b &= 0, \end{aligned} \quad (48)$$

which can only be fulfilled for $\gamma^a = \gamma^b = 90^\circ$, i.e., $\cos \gamma^a = \cos \gamma^b = 0$, if the assumption that $0^\circ \leq \gamma^a, \gamma^b \leq 90^\circ$ is made. In Fig. 19 the interaction is illustrated. Hence, we have $\mathbf{p}_v^a/p_v^a = -\mathbf{p}_v^b/p_v^b$, $\mathbf{p}_F^a/p_F^a = -\mathbf{p}_F^b/p_F^b$, $\gamma = 90^\circ$, and $\mathbf{p}_v^a \perp \mathbf{p}_F^a$, $\mathbf{p}_v^b \perp \mathbf{p}_F^b$. Now, with the aid of (7), (12), and (13) plus $\cos \gamma = 0$, one obtains

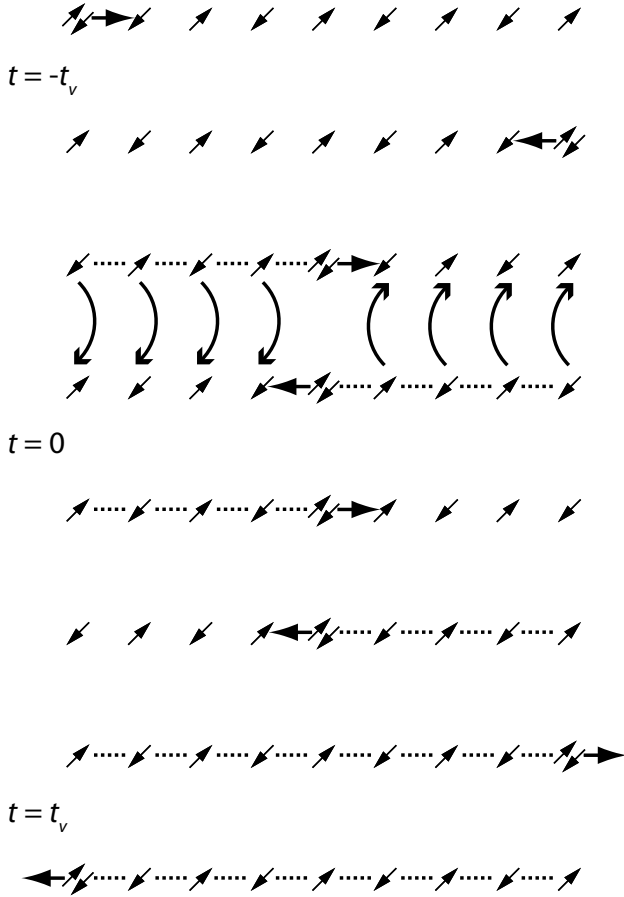


FIG. 19: Superconducting interaction between two electrons in an antiferromagnetic lattice. Every electron first performs a virtual relaxation process and, thereafter, an inverse one. Hence, it first excites magnetic moments by flipping them and, thereafter, it brings magnetic moments excited by a second electron back in their ground states by flipping them again.

$$\Delta_v = \frac{1}{2m} \pi_v^2 = \frac{1}{2} m \nu_v^2 \quad (49)$$

$$= \frac{1}{8m} p_v^2 = \frac{1}{4} E_v, \quad (50)$$

and conditions (1) and (2) become

$$\Delta_v \tau_v = \frac{\hbar}{2}, \quad (51)$$

$$\pi_v \xi_v = \hbar, \quad (52)$$

and the quantities concerning space become

$$\xi_v = \frac{\hbar}{\sqrt{2m\Delta_v}}, \quad (53)$$

$$\lambda_v = \frac{\hbar v_F}{2\Delta_v}, \quad (54)$$

where it is assumed that $\bar{v} = v_F$ in (16). It should be noted that the strength of the interaction is independent

of the distance between the electrons. And that there exists a permanent bond between two electrons which is in contrast to the picture developed so far in which electrons constantly interact with other electrons. The bond has the appearance: By performing a virtual relaxation process an electron changes its direction of propagation by

$$\Delta\alpha = \arctan \frac{2\nu_v}{v_F} = \arctan \frac{\sqrt{8\Delta_v/m}}{v_F}, \quad (55)$$

where $\Delta_v = m\nu_v^2/2$, as one can make oneself clear from Figs. 2 and 6. Thus, by performing $N = 360^\circ/\Delta\alpha$ relaxation processes it changes it by 360° and moves through one orbit. Since two electrons moving on the opposite side of such an orbit fulfill all rules for the interaction, they build a permanent bond. In Fig. 20 this configuration is illustrated. To estimate the distance between the

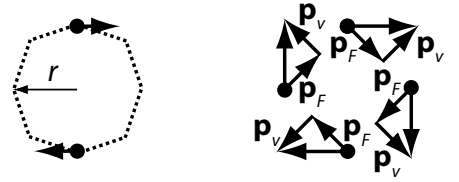


FIG. 20: Configuration in which two electrons exhibit a permanent bond for $p_v^a = p_v^b = p_v = p_F^a = p_F^b = p_F$ and $\gamma = 90^\circ$.

electrons, the orbit is approximated to a circular orbit with the radius r . Since each relaxation process moves the electron forward the distance λ_v given by (54), one gets

$$2\pi r = N\lambda_v = \frac{360^\circ}{\Delta\alpha} \lambda_v, \quad (56)$$

which can be rearranged to

$$r = \frac{1}{2\pi} \frac{360^\circ}{\Delta\alpha} \frac{\hbar v_F}{2\Delta_v}. \quad (57)$$

In Table III the radius for different normal metals is calculated. As can be seen the radius takes on large values which could be the reason why the bonds have not come to attention yet. Furthermore, because of the higher complexity of virtual processes in the antiferromagnetic lattice, these bonds are not investigated. Third, Rule III is examined. It says that Δ_0 can be calculated by

$$\Delta_0 = \frac{1}{4} E_0 W, \quad (58)$$

where (50) is used and W is the probability of finding two points which exhibit no conjured up virtual particle and at which a virtual particle can thus be created or

TABLE III: Change of the direction of propagation $\Delta\alpha$ through a relaxation process given by (55) and the radius r of the bond given by (57) for $\Delta_v(T = 0\text{ K}) = \Delta_0 = 3k_B\Upsilon_0/2$.

Normal metal	Experiment ^a		Theory	
	Υ_0 (K)	v_F (10^6 ms^{-1})	$\Delta\alpha$ ($^\circ$)	r (10^{-3} m)
Cd	0.56	1.62	0.36	1.18
Zn	0.875	1.83	0.39	0.77
Ga	1.091	1.92	0.42	0.61
Al	1.196	2.03	0.42	0.59
Tl	2.39	1.69	0.71	0.15
In	3.4	1.74	0.82	0.09
Sn	3.72	1.9	0.78	0.09
Hg	4.15	1.58	1	0.06
Pb	7.19	1.83	1.13	0.03

^aN. W. Ashcroft and D. N. Mermin, *Festkörperphysik* (Oldenbourg, München, 2007).

annihilated.¹⁴ Hence, W takes on values $0 \leq W \leq 1$. To begin with, with the normal metals is dealt. Then, (58) can be written with (23) and (24) as

$$\Upsilon_0 = \frac{1}{4} \left(\frac{k_B A}{165\hbar} \right)^{\frac{1}{4}} W \Theta^{\frac{5}{4}}, \quad (59)$$

$$\Delta_0 = \frac{3}{8} k_B \left(\frac{k_B A}{165\hbar} \right)^{\frac{1}{4}} W \Theta^{\frac{5}{4}}. \quad (60)$$

Furthermore, it is assumed that the quantities A and W are independent of the mass M of a lattice ion which means that (59) predicts closely the isotope effect $\Upsilon_0 \propto \Theta \propto M^{-1/2}$ observed in normal metals. Now, the magnitude of W is estimated which is carried out by means of the expressions (3), (21), and (25) which all describe the relaxation time. Unfortunately, about the relaxation time and which features of it determine its magnitude and thereby the magnitude of the electrical resistivity is no statement made in the literature, e.g., which is the role of the Debye temperature Θ ? A circumstance that indicates that Θ does not determine the magnitude, i.e., $\Theta \approx \text{const}$ for all normal metals, is that one observes that a large resistivity is connected with a large value of Υ_0 which is in contrast to the picture that it is small for large values of Θ . This is why it is assumed that $\Theta \approx \text{const}$.¹⁵ Now, I assume that $\bar{N} = 1$ and $\bar{E} \propto T^6/\Theta^5$ which converts function (3) into (21). Hence, it is assumed that the temperature dependence of the relaxation time results from the temperature dependence of $\bar{E}$ and that their different magnitudes result from different scattering frequencies ν_s . And, because it is thought that ν_s is proportional to W , combining (3) and (25) yields

$$W = C\rho n, \quad (61)$$

where C is a constant fitted to the experiment and is

TABLE IV: Experimental values of Υ_0 , ρ , and n plus theoretical of Υ_0 given by (62).

Normal metal	Υ_0 (K)	Experiment ^a		Theory ^b	
		ρ ($10^{-8}\text{ }\Omega\text{m}$)	n (10^{28} m^{-3})	T_0 (K)	Υ_0 (K)
Cd	0.56	1.6	9.27	50	2
Zn	0.875	1.1	13.2	51	2
Ga	1.091	2.75	15.4	39	5
Al	1.196	0.3	18.1	65	1
Tl	2.39	3.7	10.5	40	5
In	3.4	1.8	11.5	46	3
Sn	3.72	2.1	14.8	42	4
Hg	4.15	5.8	8.65	37	6
Pb	7.19	4.7	13.2	35	7

^aN. W. Ashcroft and D. N. Mermin, *Festkörperphysik* (Oldenbourg, München, 2007).

^bValues of T_0 are taken from Table II.

chosen to be $C = 1.3 \times 10^{-22} \Omega^{-1}\text{m}^2$. Then, inserting (61) into (59) and using (23) yields

$$\Upsilon_0 = \frac{1}{4} T_0 C \rho n. \quad (62)$$

In Table IV experimental are compared with theoretical values of Υ_0 . As can be seen both values are in acceptable agreement. For the antiferromagnetic lattice, W is identified with the probability of finding two hops in the direction α_{90° which cost no energy. The probability of finding one such hop is $1 - P_{gg}$, where P_{gg} is given by (39), because with the probability P_{gg} a virtual particle is conjured up, and W becomes

$$W = (1 - P_{gg})^2 \quad (63)$$

$$= [1 - (1 - \bar{N}x)^2]^2 = (2\bar{N}x - \bar{N}^2x^2)^2. \quad (64)$$

And, by combining (43), (50), (58), and (64), one gets

$$\Delta_0 = \sqrt{\frac{\hbar \Xi_x v_F}{8a}} \left[\cos \alpha + \frac{\sin \alpha}{\tan(45^\circ - \alpha)} \right]^{-\frac{1}{2}} \times (1 - \bar{N}x)(2\bar{N}x - \bar{N}^2x^2)^2. \quad (65)$$

In expression (65) all information to Δ_0 can be found. Unfortunately, I did not find values for Ξ_x and v_F , whereas the lattice constant a is well-known, which makes it impossible to predict precise values for Δ_0 and E_0 . Nevertheless, the orders of magnitude can be estimated: Because Ξ_x reflects the strength of the antiferromagnetic coupling, it is chosen to be $\Xi_x \approx 100\text{ meV}$. Furthermore, it is chosen that $v_F \approx 10^6\text{ ms}^{-1}$ and $a \approx 5 \times 10^{-10}\text{ m}$. Then, with (43) and $T_0 \approx E_0/5k_B$,¹⁶ one has

$$E_{max} = E_0(\alpha = 0^\circ, x = 0) \approx 500\text{ meV}, \quad (66)$$

$$T_{max} = T_0(\alpha = 0^\circ, x = 0) \approx 1000\text{ K}, \quad (67)$$

and for $\bar{N}$ given by (41)

$$\bar{N} \approx 3.5. \quad (68)$$

In addition, with (65) and $\Upsilon_0 \approx \Delta_0/5k_B$ plus knowing that Δ_{max} is achieved at $x \approx 0.55/\bar{N} \approx 0.16$ determined by $\partial\Delta_0/\partial x = 0$, one has

$$\Delta_{max} = \Delta_0(\alpha = 0^\circ, x = 0.16) \approx 50 \text{ meV}, \quad (69)$$

$$\Upsilon_{max} = \Upsilon_0(\alpha = 0^\circ, x = 0.16) \approx 100 \text{ K}, \quad (70)$$

which is the order of magnitude expected from the experiment. For a constant doping level, (65) may be expressed as

$$\frac{\Delta_0}{\Delta_{max}} = \left[\cos \alpha + \frac{\sin \alpha}{\tan(45^\circ - \alpha)} \right]^{-\frac{1}{2}}, \quad (71)$$

where $\Delta_0(\alpha = 0^\circ) = \Delta_{max}$, to obtain the angle dependence. In Fig. 21 function (71) is compared with the experiment. Furthermore, with (43) and (65), one gets

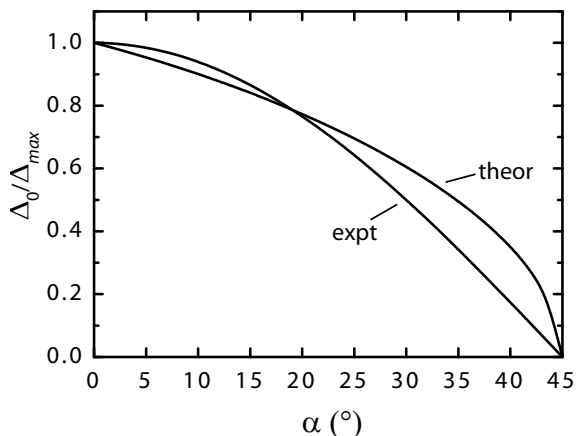


FIG. 21: Binding energy Δ_0 to its value Δ_{max} at $\alpha = 0^\circ$ vs α . The theoretical curve is given by (71) and the experimental by $\Delta_0/\Delta_{max} = \cos(2\alpha)$.

$$\Delta_0 = \frac{1}{4} E_0 (2\bar{N}x - \bar{N}^2 x^2)^2 \quad (72)$$

for the doping dependence. In Fig. 22 the doping dependences of Δ_0 and E_0 are depicted. The diagram exhibits both central features of a phase diagram of an electron system in an antiferromagnetic lattice, firstly, an increasing formation of a superconducting state with increasing the doping level until a maximum value is reached after which it decreases and vanishes and, secondly, the emergence of a second state which is well-developed at low doping levels.¹⁷ The second state is in the investigations made so far called with the expression

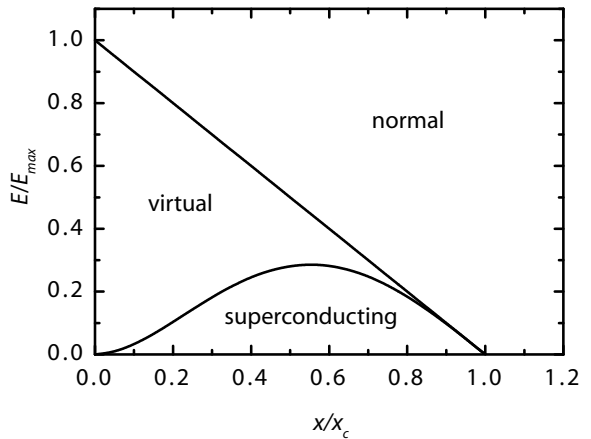


FIG. 22: Phase diagram of the electron system in an antiferromagnetic lattice. Against the x -axis the doping level x to its value x_c at $\Delta_0 = E_0 = 0$ given by $x_c = 1/\bar{N}$ is plotted. Against the y -axis the excitation energy E to its value E_{max} at $x = 0$ in (43) is plotted. The superconducting state is enclosed by (72), where for illustrative reasons it is multiplied by 4. The state in which the excitation energy takes on the constant value E_0 , which is called “virtual,” is enclosed by (43). The remaining part consists of the normal state.

“pseudogap.” Hence, the proposal for the nature of the pseudogap is that it describes that in certain systems at temperatures $\Upsilon_0 < T < T_0$ a particle takes on the constant excitation energy E_0 instead of that its excitation energy E decreases with decreasing T . And, this state is called the “virtual” state. Next, the temperature dependences are examined. The new concept predicts no new effects for $T > T_0$. For $\Upsilon_0 < T < T_0$, it predicts that electrons take on the constant excitation energy E_0 , i.e., the constant temperature T_0 . And, for $T < \Upsilon_0$, they are in a superconducting state in which they interact with each other. For the temperature-dependent virtual energies we have by means of (27) and (28)

$$E_v = E_0 - E_t = E_0 \left[1 - \frac{P_t}{P_0} \right], \quad (73)$$

$$\Delta_v = \Delta_0 - \Delta_t = \Delta_0 \left[1 - \frac{P_t}{P_0} \right], \quad (74)$$

where P_t is the probability for thermal and P_0 for virtual scattering. Then, an expression analog to (29) is formulated

$$\frac{\Delta_v}{\Delta_0} = \left[1 - \left(\frac{T}{\Upsilon_0} \right)^5 \right] \quad (75)$$

for the normal metals.¹⁸ In Fig. 23 function (75) is compared with the experiment. As can be seen both functions are in good agreement. The expected temperature dependence of the virtual momenta is illustrated in Fig. 24 and is discussed in the following. To begin with,

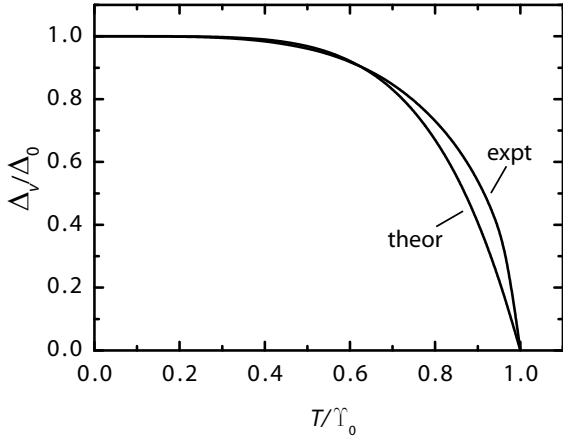


FIG. 23: Binding energy Δ_v to its value Δ_0 at $T = 0$ K vs T/Υ_0 for normal metals. The theoretical curve is given by (75) and the experimental by $\Delta_v/\Delta_0 = \{\cos[90^\circ(T/\Upsilon_0)^2]\}^{1/2}$.

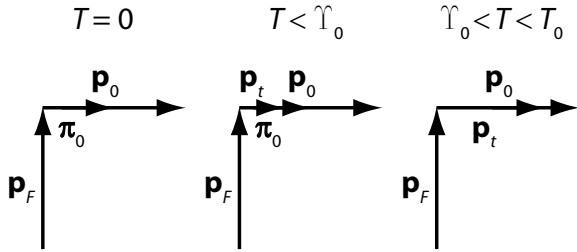


FIG. 24: Temperature dependence of the virtual momenta for $\gamma = 90^\circ$.

expressions analog to (73) and (74) are formulated

$$\mathbf{p}_v = \mathbf{p}_0 - \mathbf{p}_t, \quad (76)$$

$$\boldsymbol{\pi}_v = \boldsymbol{\pi}_0 - \boldsymbol{\pi}_t. \quad (77)$$

Furthermore, the thermal momenta $\mathbf{p}_t$ are treated in the same way as the negative virtual were treated which implies that

$$E_t = \frac{1}{2m} \mathbf{p}_t^2, \quad (78)$$

and by taking the scalar form of (76) and (77) plus using (73), (74), $E_0 = p_0^2/2m$, and $\Delta_0 = \pi_0^2/2m$, one obtains

$$p_v = \sqrt{2mE_0} \left[1 - \sqrt{\frac{P_t}{P_0}} \right], \quad (79)$$

$$\pi_v = \sqrt{2m\Delta_0} \left[1 - \sqrt{\frac{P_t}{P_0}} \right]. \quad (80)$$

And, with the aid of (75), the negative virtual momentum for normal metals becomes

$$\pi_v = \sqrt{2m\Delta_0} \left[1 - \left(\frac{T}{\Upsilon_0} \right)^{\frac{5}{2}} \right]. \quad (81)$$

As a conclusion all insights are summarized in Fig. 25.

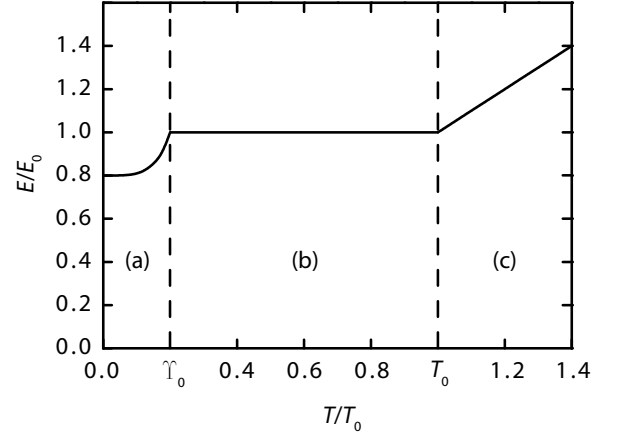


FIG. 25: Excitation energy E of an electron to its minimum excitation energy E_0 vs T/T_0 for $E_0/\Delta_0 = T_0/\Upsilon_0 = 5$. In the superconducting state (a), electrons possess the energy $E_0 - \Delta_v$ and interact with the strength Δ_v . In the virtual state (b), they possess the constant excitation energy E_0 . In the normal state (c), no new effects are predicted, i.e., E increases with increasing T .

At last, I make one remark. In the treatment made the repulsive Coulomb interaction

$$F_C = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \quad (82)$$

between electrons is not taken into account. Thus, to my mind, it can be neglected which is in contrast to the picture developed so far in which it makes a considerable counterpart to the attractive superconducting interaction. Its neglecting is attributable to the neglecting of electron-electron interactions in the derivation of the excitation-energy-dependent relaxation time, e.g., in normal metals they are neglected compared to electron-phonon interactions. Besides, it seems strange that, on the one hand, in the relaxation time electron-electron are negligible compared to electron-phonon interactions and, on the other hand, in a superconductor they are not.

B. Perfect conductivity

Perfect conductivity is the hallmark of superconductivity. Its observation led to the realization that systems may exhibit a superconducting state and after it superconductivity was named. Therefore, a theory of superconductivity must contain an explanation for it. Unfortunately, this subject is much disregarded which shows

itself in the circumstance that there does not exist one common explanation, where in many works to superconductivity the subject is not even treated. This is an unsatisfactory and sad situation which, to my mind, reflects that the heart of superconductivity is still not revealed. In the following, I present an explanation which hopefully sheds light on it.

A requirement to understand superconductivity is seen to be to understand the electrical resistivity, which is in the Drude model described by the formula

$$\rho = \frac{m}{ne^2t}, \quad (83)$$

where m is the mass and e the charge of an electron, n is the density of electrons, and t the relaxation time. Because the quantities m , e , and n take on constant values, the relaxation time t is used to explain the perfect conductivity, i.e., the vanishing of the resistivity, $\rho = 0$. This work says that the relaxation time describes the period of time in which an electron loses its excitation energy E , which results from all mechanisms responsible for a gain of energy, e.g., the thermal energy and the electric energy due to an applied electric field. Thus, because an electron exhibits the energy E , it experiences processes responsible for a loss of electric energy, i.e., a decrease of the electric current and a finite resistivity. However, in the superconducting state, the resistivity vanishes which is explained as follows: Up to now, we talked about real relaxation processes. But in the superconducting state virtual relaxation processes must also be taken into account. The point I use to explain perfect conductivity is that the interaction takes place in the form that, first, a virtual and, thereafter, an inverse relaxation process is performed. Within the inverse all within the virtual relaxation process created particles have to be annihilated in inverse order so that the electron goes through a series of allowed states. This holds also for created real particles. But since they are in general not in the position to be annihilated, a difficulty opens up which is circumvented by assuming that no real particles can be created within a virtual relaxation process. Therefore, one gets that an electron is only able to perform either virtual or real relaxation processes. In the superconducting state, only virtual and no real relaxation processes are performed which implies that the real relaxation time goes to infinity and the electrical resistivity vanishes. Hence, perfect conductivity becomes a condition that the superconducting interaction can take place.

At last, I draw the attention to the fact that the relaxation time is interpreted differently as it was so far. Following from this, in Appendix B new insights to the electrical resistivity are presented, such as, an explanation for Ohm's law and a new limit for its range of validity.

C. Energy gap

Another feature of superconductivity is the emergence of an energy gap. It describes the gap between the energy levels occupied by electrons (holes) in the bond and the energy levels reached by electrons (holes) by exciting them. The interaction leading to the bond is described by relaxation processes in which electrons give off the energy Δ_v . Thus, $2\Delta_v$ is interpreted as the energy required to break up the bond between two electrons and Δ_v becomes an energy gap. Furthermore, because of the similarities between virtual and thermal processes, the occupation probabilities of states at a given virtual energy are expected to be equal to the ones at the corresponding thermal energy. In Fig. 26 these considerations are depicted. In the picture of virtual processes one can

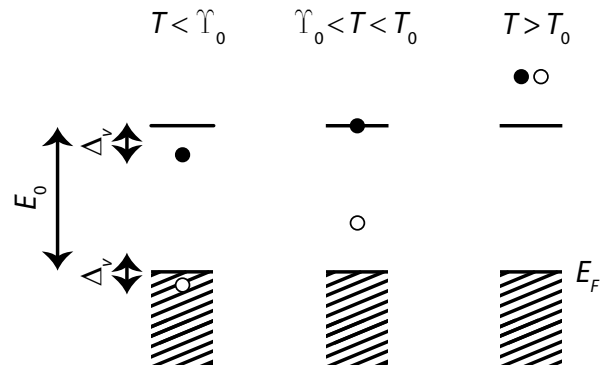


FIG. 26: Energy of an electron for different temperature ranges in the picture developed within this work ($\bullet$) and in the picture developed so far ($\circ$). The energy of the ground state is the Fermi energy E_F and states with $E \leq E_F$ are occupied. For $T < T_0$, there is the difference that Δ_v is measured in respect to $E_F + E_0$ and to $E_F + E$. For $T_0 < T < T_0$, the picture developed within this work predicts that the electron takes on a constant excitation energy E_0 , whereas the picture developed so far predicts no deviations from the normal state, i.e., the excitation energy E increases with increasing T . For $T > T_0$, both pictures coincide.

explain the energy gap also by the argument that in order to break up the bond the energy $2\Delta_v$ must be put in form of created real particles into the solid so that the virtual scattering probability becomes zero. Then, electrons no longer interact with each other via virtual relaxation processes and move independently.

Next, I deal with how the relationship between the virtual energies (E_v, Δ_v) and the virtual temperatures (T_v, Υ_v) looks like. From my point of view, it is given by the relationship between the excitation energy E and the temperature T , i.e.,¹⁹

$$\frac{E_v}{T_v} = \frac{\Delta_v}{\Upsilon_v} = \frac{E}{T}. \quad (84)$$

In classical physics the relating of T and E was examined and the equipartition relation was obtained

$$E = \frac{f}{2} k_B T, \quad (85)$$

$$\frac{E}{T} = \frac{f}{2} k_B, \quad (86)$$

where f is the number of the degrees of freedom of a particle. In case that the quantum nature of the states can be neglected (i.e., the quantum number n becomes $n \gg 1$), relation (86) holds also in quantum physics. The range of validity is estimated by Eq. (44) in which E_v is replaced by ΔE and p_v by Δp , where ΔE and Δp shall characterize the quantum nature concerning energy and momentum,

$$\Delta E = \frac{1}{2m} (2p_F \Delta p \cos \gamma + \Delta p^2). \quad (87)$$

Then Δp is chosen to be

$$\Delta p = \hbar \Delta k = \hbar \frac{2\pi}{L}, \quad (88)$$

where Δk describes the distance in $\mathbf{k}$ -space between nearest-neighbor states and L is thus the size of the sample. Therefore, ΔE becomes the energy difference between nearest-neighbor states. Now, the equipartition relation holds if ΔE is small compared to E , $\Delta E \ll E$. In normal metals, typical values are $L \approx 1$ mm and $p_F = mv_F \approx 10^{-24}$ kgms $^{-1}$. Then, with (86), (87), and (88), plus $\Delta p \ll p_F$ and $\cos \gamma \approx 1$, one obtains

$$\Delta E \approx \frac{p_F \Delta p}{m} \approx 10^{-6} \text{ eV}, \quad (89)$$

$$\Delta T \approx \frac{\Delta E}{k_B} \approx 0.01 \text{ K}. \quad (90)$$

Hence, for $T \gg 0.01$ K, the equipartition relation holds. And, for a free particle, as it is an electron in the free-electron-gas model, one gets three translational degrees of freedom $f = 3$ which implies that (84) becomes

$$\frac{E_v}{T_v} = \frac{\Delta_v}{\Upsilon_v} = \frac{E}{T} = \frac{3}{2} k_B = 1.5 k_B. \quad (91)$$

This differs from the prediction

$$\frac{\Delta_v}{\Upsilon_v}(T = 0 \text{ K}) = \frac{\Delta_0}{\Upsilon_0} = 1.764 k_B \quad (92)$$

of the BCS theory.² However, it is noted that within this work (91) fits the experiment better than (92). Furthermore, there exists the case that ΔE reaches and exceeds E , $\Delta E \gtrsim E$, where (86) no longer holds. How the relationship between T and E looks like if the quantum nature must be taken into account is not examined in detail yet. However, with the results obtained

from such examinations, one obtains predictions for (84) which can be compared with the experiment to check my point of view. For normal metals, ΔE was estimated to be $\Delta E \approx 10^{-6}$ eV ($\Delta T \approx 0.01$ K) for sample sizes $L \approx 1$ mm. If L is reduced to $L \ll 1$ mm, though, a deviation of (84) from $1.5 k_B$ appears at higher temperatures. Actually, the two-dimensional CuO₂ plane is a system in which the equipartition relation should not be valid because the third dimension is only of the order of a lattice constant, $L \approx 1$ nm $\ll 1$ mm. This behavior is already observed in cuprate superconductors²⁰ in which Δ_0/Υ_0 is bigger than $1.5 k_B$. Furthermore, if point of view (84) is true, a new approach to describe (cuprate) superconductors opens up. E.g., it is conceivable that different values of Υ_0 for similar values of Δ_0 can be explained by different relations (84). By that the phenomenon that Υ_0 depends on the number of CuO₂ planes in the unit cell could be explained if hopping between CuO₂ planes is assumed to determine the movement in the third dimension which determines L and ΔE , and thereby relation (84). At last, I note that there is a contrast between the picture developed within this work and the picture developed so far in respect to the answer to the question how Δ_0/Υ_0 is expected to look like for large values of Υ_0 . Whereas in the picture developed so far a large value of Δ_0/Υ_0 is connected with a strong-coupling superconductor with a large value of Υ_0 , the picture developed within this work predicts that, for a given value Δ_0 , Δ_0/Υ_0 should possess a small value in order that Υ_0 becomes large.

D. Specific heat

In this subsection the specific heat (capacity) c is examined. At first, it is assumed that only the contribution c_e of the electrons has to be newly calculated which is done in the following for the normal metals. Furthermore, because electrons (holes) in the antiferromagnetic lattice exhibit more complex virtual energies, this determination is not carried out. Nevertheless, on the basis of the results for the normal metals, general predictions can be made.

To begin with, the internal energy U of the electron system is determined. Because the virtual temperatures Υ_v and T_v and the temperature T are treated in a related way, it is first written as

$$U = \frac{\pi^2}{6} D(E_F) (k_B T)^2, \quad (93)$$

where $D(E_F)$ is the density of states at the Fermi energy.²¹ For $T > T_0$, the postulate predicts no new effects. For $\Upsilon_0 < T < T_0$, electrons take on the constant temperature T_0 which implies that the internal energy takes on the constant value

$$U = \frac{\pi^2}{6} D(E_F) (k_B T_0)^2. \quad (94)$$

And, for $T < \Upsilon_0$, they take on additionally a negative virtual temperature which is taken into account as follows

$$\begin{aligned} U &= \frac{\pi^2}{6} D(E_F) (k_B T_0)^2 - \frac{\pi^2}{6} D(E_F) (k_B \Upsilon_v)^2 \\ &= \frac{\pi^2}{6} D(E_F) k_B^2 (T_0^2 - \Upsilon_v^2), \end{aligned} \quad (95)$$

where the temperature-dependent virtual temperature is obtained with (75) and (91)

$$\Upsilon_v = \Upsilon_0 \left[1 - \left(\frac{T}{\Upsilon_0} \right)^5 \right]. \quad (96)$$

In Fig. 27 the considerations made are depicted. Now,

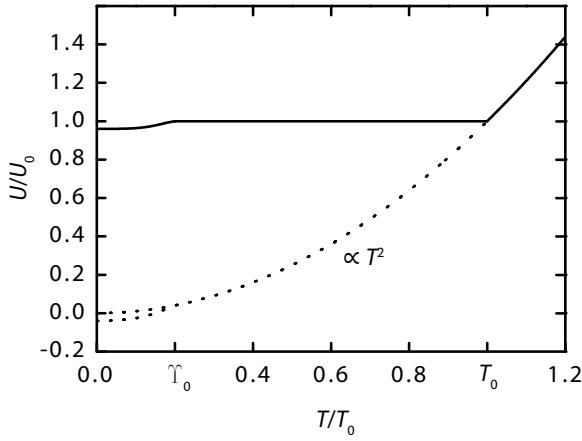


FIG. 27: Internal energy U of the electron system in a superconductor to its value U_0 given by (94) vs T/T_0 for $T_0/\Upsilon_0 = 5$. For $T < \Upsilon_0$, U is given by (95). For $\Upsilon_0 < T < T_0$, U is given by (94). For $T > T_0$, U is given by (93). The dashed lines represent the normal state given by (93) and the picture developed so far in which Υ_v is measured in respect to (93).

to get a new perspective, the internal energy (95) is re-expressed with the aid of (91)

$$U = \frac{2\pi^2}{27} D(E_F) (E_0^2 - \Delta_v^2), \quad (97)$$

where the second term

$$U_{cond} = -\frac{2\pi^2}{27} D(E_F) \Delta_v^2 \quad (98)$$

$$\approx -0.73 D(E_F) \Delta_v^2 \quad (99)$$

describes the energy given off by the electron system and is interpreted as a condensation energy. And, it is noted that (98) is similar to the expression

$$U_{cond} = -\frac{1}{4} D(E_F) \Delta_0^2 \quad (100)$$

predicted by the BCS theory² for the condensation energy at $T = 0$ K. Next, to obtain the specific heat c_e in a normal metal, U is differentiated with respect to T ,

$$c = \frac{\partial}{\partial T} U. \quad (101)$$

This yields for $T > T_0$ and U given by (93)

$$c_e = \frac{\pi^2}{3} D(E_F) k_B^2 T, \quad (102)$$

for $\Upsilon_0 < T < T_0$ and U given by (94)

$$c_e = 0, \quad (103)$$

and for $T < \Upsilon_0$, U given by (95) and Υ_v by (96)

$$\begin{aligned} c_e &= -\frac{\pi^2}{6} D(E_F) k_B^2 \frac{\partial}{\partial T} \Upsilon_v^2 \\ &= -\frac{\pi^2}{3} D(E_F) k_B^2 \Upsilon_v \frac{\partial}{\partial T} \Upsilon_v \\ &= \frac{\pi^2}{3} D(E_F) k_B^2 \Upsilon_0 \\ &\quad \times 5 \left[\left(\frac{T}{\Upsilon_0} \right)^4 - \left(\frac{T}{\Upsilon_0} \right)^9 \right]. \end{aligned} \quad (104)$$

In Fig. 28 the temperature-dependent electronic specific heat is shown. As the reader may have noticed, there

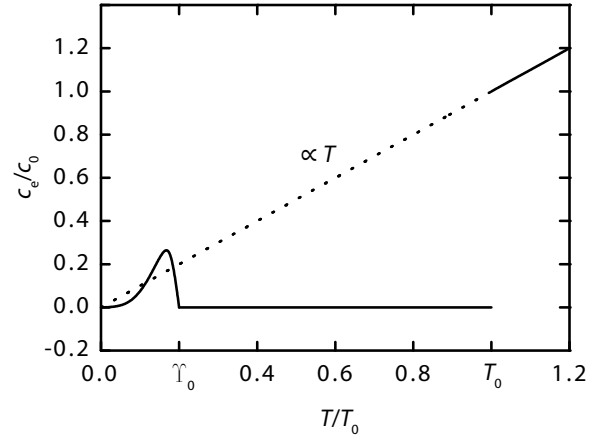


FIG. 28: Specific heat c_e of the electron system in a superconductor to the value c_0 given by (102) at $T = T_0$ vs T/T_0 for $T_0/\Upsilon_0 = 5$ (cf. Fig. 27). For $T < \Upsilon_0$, c_e is given by (104). For $\Upsilon_0 < T < T_0$, c_e is given by (103). For $T > T_0$, c_e is given by (102). The dashed line represents the normal state (102).

appear differences between these results and the behavior expected from the picture developed so far. At first, it should be said that it is challenging to determine $c_e(T)$

experimentally because the contribution of the electrons is in general small compared to others, e.g., the specific heat of phonons. This is why $c_e(T)$ may still be in part unknown and new predictions may still turn out to be true. Now, the differences are discussed. Firstly, for $\Upsilon_0 < T < T_0$, the postulate predicts the virtual state in which the internal energy takes on a constant value, $c_e = \partial U / \partial T = 0$, whereas the picture developed so far predicts no deviations from the normal state. Also, at $T = T_0$, a discontinuity occurs in form of a jump from $c_e = 0$ to $c_e = \pi^2 D(E_F) k_B^2 T_0 / 3$. Secondly, I discuss the temperature range $T < \Upsilon_0$ which is additionally shown in Fig. 29. For small temperatures the contribution of

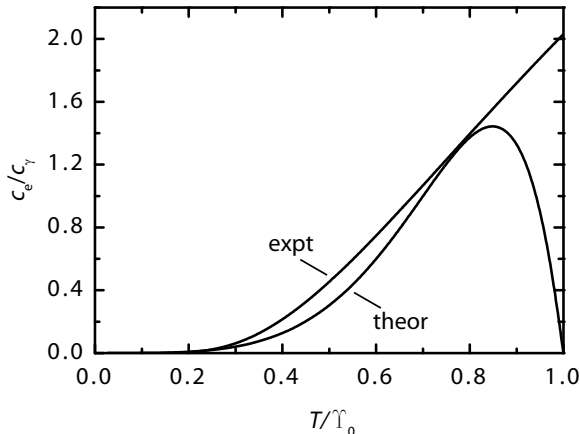


FIG. 29: Specific heat c_e of the electron system in a superconductor given by (104) to the value c_γ given by (102) at $T = \Upsilon_0$ vs T/Υ_0 . The experimental curve is given by²² $c_e/c_\gamma = 9.1 \exp(-1.5\Upsilon_0/T)$.

the electrons is expected to become less small compared to others which makes it easier to determine $c_e(T)$. Such examinations showed that for $T \ll \Upsilon_0$ it takes on the form

$$\frac{c_e}{c_\gamma} = a e^{-b \frac{\Upsilon_0}{T}}, \quad (105)$$

where c_γ is (102) at $T = \Upsilon_0$ [$c_\gamma = \pi^2 D(E_F) k_B^2 \Upsilon_0 / 3$], and a and b are numerical constants. Experiments by Corak *et al.*²² produced $a = 9.10$ and $b = 1.50$. As can be seen the experimental specific heat (105) is, in its range of validity, in acceptable agreement with the theoretical specific heat

$$\frac{c_e}{c_\gamma} = 5 \left[\left(\frac{T}{\Upsilon_0} \right)^4 - \left(\frac{T}{\Upsilon_0} \right)^9 \right] \quad (106)$$

obtained from (104). However, for $T \lesssim \Upsilon_0$, there appear bigger differences. In the picture developed so far the specific heat in the superconducting state is expected to show a qualitative behavior in form of function (105),

i.e., c_e increases with increasing T and at $T = \Upsilon_0$ a discontinuity occurs in form of a jump to c_γ . The picture developed within this work, again, predicts that c_e given by (106) increases with increasing T only for $0 < T < T_{max}$, where at $T = T_{max}$ it takes on its maximum value and thus T_{max} can be calculated by

$$\begin{aligned} \frac{\partial}{\partial T} c_e &= 0 \\ 4 \left(\frac{T_{max}}{\Upsilon_0} \right)^3 - 9 \left(\frac{T_{max}}{\Upsilon_0} \right)^8 &= 0 \\ 4 - 9 \left(\frac{T_{max}}{\Upsilon_0} \right)^5 &= 0, \end{aligned}$$

which yields

$$T_{max} = \left(\frac{4}{9} \right)^{\frac{1}{5}} \Upsilon_0 \quad (107)$$

$$\approx 0.85 \Upsilon_0. \quad (108)$$

And, for $T_{max} < T < \Upsilon_0$, it decreases with increasing T . To my mind, the reason for the difference near Υ_0 is: The experiment is not interpreted correctly. The mistake is made that the rapid fall to zero between T_{max} and Υ_0 is interpreted as an instant jump. This work says though that there exists a range in which c_e shows a shoulder-like behavior characterized by a decrease and that this range is part of the superconducting state and no discontinuity occurs at $T = \Upsilon_0$. As a conclusion to this paragraph, I derive the BCS value $1.764 k_B$. The BCS theory predicts a specific heat in the qualitative form of function (105), where in my opinion, the situation is: The value of Υ_0 in the BCS theory Υ_0^{BCS} is not chosen to be Υ_0 at which the superconducting state sets in, but it is chosen to be a lower value $\Upsilon_0^{\text{BCS}} < \Upsilon_0$ so that a jump of c_e at $T = \Upsilon_0^{\text{BCS}}$ is predicted. The obvious choice for Υ_0^{BCS} within this work is T_{max} given by (107)

$$\Upsilon_0^{\text{BCS}} = T_{max} = \left(\frac{4}{9} \right)^{\frac{1}{5}} \Upsilon_0. \quad (109)$$

Now, with the aid of (91), we have

$$\frac{\Delta_0}{\Upsilon_0} = \left(\frac{4}{9} \right)^{\frac{1}{5}} \frac{\Delta_0}{\Upsilon_0^{\text{BCS}}} = \frac{3}{2} k_B, \quad (110)$$

which implies that

$$\frac{\Delta_0}{\Upsilon_0^{\text{BCS}}} = \left(\frac{9}{4} \right)^{\frac{1}{5}} \frac{3}{2} k_B \quad (111)$$

$$\approx 1.764 k_B, \quad (112)$$

which is the BCS value (92). The excellent agreement makes it hard to believe that the derivation is not justified. And, it supports the validity of (104) as well as

the point of view that Δ_0/Υ_0 takes on the value $1.5k_B$ instead of $1.764k_B$.

At last, predictions about the electronic specific heat in general are made. Firstly, electrons in a superconductor are expected to exhibit a virtual state in which the specific heat vanishes and a discontinuity occurs at $T = T_0$. Secondly, in the superconducting state, the specific heat may possess a form with an increase and a decrease and in which no discontinuity occurs at $T = \Upsilon_0$, which is in contrast to the picture developed so far in which the discontinuity is seen to be an important feature. This shoulder-like behavior is already observed in cuprate superconductors.²³ However, in the picture developed so far the shoulder is seen to exist for $T > \Upsilon_0$, whereas the picture developed within this work says that it is part of the superconducting state.

E. Superconductor in a magnetic field

At last, I examine a superconductor in a magnetic field $\mathbf{B}$. Its presence means that electrons (holes) see, in addition to the superconducting, a magnetic interaction which is described by the Lorentz force

$$\mathbf{F}_L = e(\mathbf{v} \times \mathbf{B}). \quad (113)$$

In the following, it is assumed that the velocity $\mathbf{v}$ which is identified with (16) can be set equal to the Fermi velocity $\mathbf{v}_F$, $\mathbf{v}_g = \mathbf{v}_F$ and $v_v \ll v_F$, and (113) becomes

$$\mathbf{F}_L = e(\mathbf{v}_F \times \mathbf{B}). \quad (114)$$

Next, the interplay between the superconducting and the magnetic interaction is investigated. Hence, an expression for the superconducting force is formulated

$$\mathbf{F}_s = \frac{\Delta_v}{\xi_v} \mathbf{e}_\xi = \frac{\pi_v}{\tau_v} \mathbf{e}_\pi, \quad (115)$$

where $\mathbf{e}_\xi$ is the unit vector in the direction of ξ_v and $\mathbf{e}_\pi$ the one in the direction of π_v . Its justification is found in the perspective of relaxation processes. Because a force describes the change of energy (momentum) in a certain direction within a certain distance (within a certain period of time), and because the energy Δ_v (the momentum π_v) describes the mean change of energy (momentum) by performing a virtual relaxation process in the direction $\mathbf{e}_\xi = \mathbf{e}_\pi$ within the distance ξ_v (within the period of time τ_v), it seems obvious to formulate (115). Now, the total force acting on an electron in a superconductor in a magnetic field becomes

$$\begin{aligned} \mathbf{F} &= \mathbf{F}_s + \mathbf{F}_L \\ &= \frac{\Delta_v}{\xi_v} \mathbf{e}_\xi + e(\mathbf{v}_F \times \mathbf{B}). \end{aligned} \quad (116)$$

In the following investigation, the scalar form is taken

$$F = \frac{\Delta_v}{\xi_v} + ev_F B \sin \beta, \quad (117)$$

where β is the angle between $\mathbf{v}_F$ and $\mathbf{B}$. In analogy to the influence of the temperature, a magnetic field is expected to lead to additional scattering processes. Therefore, a decrease of Δ_v becomes the consequence and the negative virtual energy is written in analogy to (27) as

$$\Delta_B = \Delta_v - \Delta_v \frac{P_B}{P_v}, \quad (118)$$

where P_B is the probability for magnetic and P_v for virtual scattering. Because magnetic scattering is characterized by the Lorentz force, P_B/P_v is written as

$$\begin{aligned} \Delta_B &= \Delta_v - \Delta_v \frac{|ev_F B \sin \beta|}{\Delta_v/\xi_v} \\ &= \Delta_v \left[1 - \frac{|ev_F B \sin \beta|}{\Delta_v/\xi_v} \right], \end{aligned} \quad (119)$$

which holds if virtual dominates magnetic scattering, $\Delta_v/\xi_v > |ev_F B \sin \beta|$. This means that the influence shows itself indirectly, i.e., electrons do not undergo a change of the state of motion described by the Lorentz force. However, as soon as magnetic dominates virtual scattering, $\Delta_v/\xi_v < |ev_F B \sin \beta|$, they experience the magnetic interaction and the bond breaks up. Next, the subject for what cause an applied magnetic field destroys the perfect conductivity is treated. This is expected to arise the moment that a part of the electrons do no longer interact via the superconducting interaction and no longer perform virtual but real relaxation processes, i.e., $\Delta_v/\xi_v < |ev_F B \sin \beta|$. Then, electrons relax in the ground state and give off the electric energy which leads to the decrease of the electric current and the finite resistivity. Since the Lorentz force takes on its maximum value for $\sin \beta = 1$, I get the following condition for the destruction of the perfect conductivity

$$\frac{\Delta_v}{\xi_v} = ev_F B_c, \quad (120)$$

where B_c is the critical magnetic field. Now, (119) can be put for $\sin \beta = 1$ to

$$\frac{\Delta_B}{\Delta_v} = \left[1 - \frac{B}{B_c} \right]. \quad (121)$$

Furthermore, after inserting $\xi_v = \hbar/\pi_v$ into (120), rearranging it yields

$$B_c = \frac{1}{\hbar ev_F} \pi_v \Delta_v. \quad (122)$$

For normal metals, we have (75) and (81), and thus we have the temperature-dependent critical field

$$B_c = \frac{\sqrt{2m}}{\hbar v_F} \left(\frac{3}{2} k_B \Upsilon_0 \right)^{\frac{3}{2}} \times \left[1 - \left(\frac{T}{\Upsilon_0} \right)^{\frac{5}{2}} \right] \left[1 - \left(\frac{T}{\Upsilon_0} \right)^5 \right] \quad (123)$$

$$\frac{B_c}{B_0} = \left[1 - \left(\frac{T}{\Upsilon_0} \right)^{\frac{5}{2}} \right] \left[1 - \left(\frac{T}{\Upsilon_0} \right)^5 \right]. \quad (124)$$

In Fig. 30 function (124) is compared with the experimental function

$$\frac{B_c}{B_0} = \left[1 - \left(\frac{T}{\Upsilon_0} \right)^2 \right]. \quad (125)$$

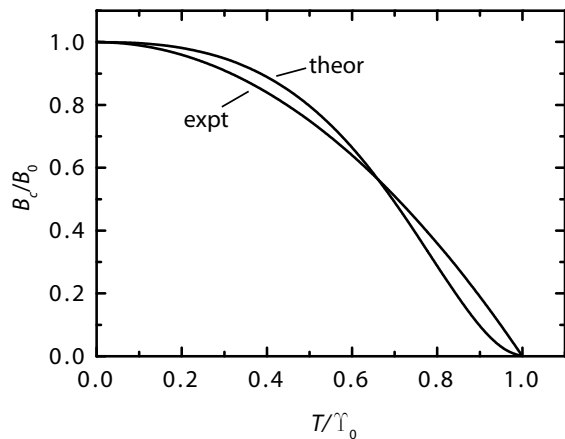


FIG. 30: Critical magnetic field B_c to its value B_0 at $T = 0$ K vs T/Υ_0 . The theoretical curve is given by (124) and the experimental by (125).

As can be seen both functions are in good agreement. And, for $T = 0$ K in (123), one obtains

$$B_c(T = 0 \text{ K}) = B_0 = \frac{\sqrt{2m}}{\hbar v_F} \left(\frac{3}{2} k_B \Upsilon_0 \right)^{\frac{3}{2}} \quad (126)$$

for the maximum critical field. In Table V for different normal metals B_0 is calculated and compared with the experiment. One can say that the agreement is good and, for $\Upsilon_0 > 2$ K, it is even excellent. In addition, I remind that the value $1.5k_B$ and not the BCS value $1.764k_B$ is used in (126) and that the chosen value predicts B_0 better which supports the point of view that Δ_0/Υ_0 takes on the value $1.5k_B$ instead of $1.764k_B$. At last, the magnetic properties are discussed. The configuration of the electron system in a superconductor in a magnetic field is the one with the minimum energy. Thus, because the

TABLE V: Experimental values of Υ_0 , v_F , and B_0 plus theoretical of B_0 given by (126).

Normal metal	Υ_0 (K)	Experiment ^a		Theory
		v_F (10^6 ms^{-1})	B_0 (10^{-3} T)	B_0 (10^{-3} T)
Cd	0.56	1.62	3.0	1.9
Zn	0.875	1.83	5.3	3.4
Ga	1.091	1.92	5.1	4.5
Al	1.196	2.03	9.9	4.9
Tl	2.39	1.69	17.1	16.5
In	3.4	1.74	29.3	27.1
Sn	3.72	1.9	30.5	28.4
Hg	4.15	1.58	41.1	40.3
Pb	7.19	1.83	80.3	79.3

^aN. W. Ashcroft and D. N. Mermin, *Festkörperphysik* (Oldenbourg, München, 2007).

presence of a magnetic field decreases Δ_B , it is expected that the electron system exhibits a configuration in which the magnetic field is expelled out of the sample, where it is expected that this is achieved by electric screening currents at the surface of the sample. This describes a perfect diamagnet which is the observed behavior for $B < B_c$. Its breakdown for $B > B_c$ is explained by the argument that as soon as electrons perform real relaxation processes and experience a resistance, the currents responsible for the screening cannot be kept up. If all electrons experience the superconducting interaction and perform virtual relaxation processes, again, no resistance occurs and they can flow eternally. The Meißner-Ochsenfeld effect, i.e., the expulsion of the magnetic field in the superconducting state takes place independent of how the state is reached, is seen to be also predicted because superconductivity is described by the emergence of a new interaction. This is so because the configuration which the electron system exhibits is determined by the properties of the electrons and the interactions between them and not by the history for what cause they emerge.

Up to now, we examined one side of the coin of a superconductor in a magnetic field, the type I superconductors. However, there exists a second side, the type II superconductors which are examined next.

In a type I superconductor, electrons do not undergo a change of the state of motion due to the Lorentz force. However, it is imaginable that there exists a scenario in which they do undergo it and simultaneously interact with each other via the superconducting interaction. The occurrence of this second scenario is seen to mean that electrons are in a type II superconductor, where its appearance is seen to be: The decisive point is that it is expected that the ground state predetermined at the beginning of the virtual relaxation process must be equal to the ground state reached after performing it, i.e., at the beginning an electron possesses the energy $E_g + E_v$ and the momentum $\mathbf{p}_g + \mathbf{p}_v$ and at the end it must possess the energy E_g and the momentum $\mathbf{p}_g$. Therefore, an additional force is in general not allowed to act because

it would change the ground state during the relaxation process. Nevertheless, there exists a scenario in which it is allowed to act. Namely, if it acts in a way that after performing the virtual relaxation process the electron is again in its initial ground state. In a magnetic field, this situation is achievable because an electron experiencing the Lorentz force is moving in a circular orbit with the cyclotron radius r_c and after running through the orbit once it is again in its initial state. Hence, if the virtual relaxation process is performed during one such run, an electron is able to experience both forces (114) and (115) simultaneously. Next, an expression for Δ_B in the second scenario is formulated by assuming that the scattering probability is proportional to the inverse relaxation time t^{-1} , which converts (118) into

$$\Delta_B = \Delta_v - \Delta_v \frac{\tau_v}{t_B}, \quad (127)$$

where t_B is the magnetic relaxation time. Furthermore, since the relaxation time is the relaxation length divided by the mean velocity, $t = l/\bar{v}$, and $\bar{v}$ is the same for both relaxation times, (127) is equivalent to

$$\Delta_B = \Delta_v - \Delta_v \frac{\lambda_v}{2\pi r_c}, \quad (128)$$

where the magnetic relaxation length is chosen to be the circumference of the circular orbit. By equating the Lorentz and the centripetal force one gets

$$r_c = \frac{mv}{eB}. \quad (129)$$

And, with the quantum condition

$$mvr = n\hbar, \quad (130)$$

in which r is identified with r_c , one gets

$$r_c = \sqrt{n \frac{\hbar}{eB}}, \quad (131)$$

where $n = 1, 2, 3, \dots$. Finally, with (54), (128), and (131), one has

$$\Delta_B = \Delta_v \left[1 - \frac{\hbar v_F}{4\pi \Delta_v} \sqrt{\frac{eB}{n\hbar}} \right], \quad (132)$$

where I expect that (132) is only valid for $n = 1$. This is so because the smallest cyclotron radius is seen to be connected with $\sin \beta = 1$, which is used for the relaxation length ($\bar{v}$ is chosen to be v_F and not $v_F \sin \beta$), and (132) becomes

$$\Delta_B = \Delta_v \left[1 - \frac{\hbar v_F}{4\pi \Delta_v} \sqrt{\frac{eB}{\hbar}} \right]. \quad (133)$$

Now, because electrons want to minimize their energy by maximizing Δ_B , they exhibit the first scenario if (119) exceeds (133) and the second if (133) exceeds (119). The condition for the equality of (119) and (133) is

$$\Delta_v \left[1 - \frac{ev_F B_{c1}}{\Delta_v \xi_v} \right] = \Delta_v \left[1 - \frac{\hbar v_F}{4\pi \Delta_v} \sqrt{\frac{eB_{c1}}{\hbar}} \right], \quad (134)$$

where $\sin \beta = 1$ is used which yields with $\xi_v = \hbar/\pi v$

$$B_{c1} = \frac{1}{16\pi^2 \hbar e} \pi_v^2, \quad (135)$$

where B_{c1} is the lower critical magnetic field and for $B < B_{c1}$ electrons exhibit the first scenario [Δ_B given by (119) for $\sin \beta = 1$ exceeds Δ_B given by (133)]. At $T = 0$ K, one has $\Delta_0 = \pi_0^2/2m$ and one gets

$$B_{c1}(T = 0 \text{ K}) = B_{01} = \frac{m}{8\pi^2 \hbar e} \Delta_0. \quad (136)$$

And, there exists an upper critical magnetic field B_{c2} if Δ_B given by (128) and (133) becomes zero

$$\Delta_v \left[1 - \frac{\lambda_v}{2\pi r_c} \right] = 0, \quad (137)$$

which yields

$$\lambda_v = 2\pi r_c \quad (138)$$

$$\frac{\hbar v_F}{2\Delta_v} = 2\pi \sqrt{\frac{\hbar}{eB_{c2}}} \quad (139)$$

and, finally,

$$B_{c2} = \frac{16\pi^2}{\hbar e v_F^2} \Delta_v^2. \quad (140)$$

For $B_{c1} < B < B_{c2}$, electrons exhibit the second scenario in which Δ_B is given by (133). At $T = 0$ K, one gets

$$B_{c2}(T = 0 \text{ K}) = B_{02} = \frac{16\pi^2}{\hbar e v_F^2} \Delta_0^2. \quad (141)$$

And, with the aid of (140), (133) can be put to

$$\frac{\Delta_B}{\Delta_v} = \left[1 - \sqrt{\frac{B}{B_{c2}}} \right]. \quad (142)$$

Now, a superconductor is of type II if its critical field B_{02} exceeds its critical field B_0 because then electrons exhibit, for $B_{c1} < B < B_{c2}$, the second scenario. Thus, the condition for type II superconductivity goes

$$B_{02} > B_0, \quad (143)$$

which yields with (126) and (141)

$$\frac{16\pi^2}{\hbar v_F^2} \Delta_0^2 > \frac{\sqrt{2m}}{\hbar v_F} \Delta_0^{\frac{3}{2}}, \quad (144)$$

which can be rearranged to

$$\frac{128\pi^4 \Delta_0}{m v_F^2} > 1. \quad (145)$$

With the aid of (145), the minimum value Υ_0^{min} for type II superconductivity can be estimated if one takes the typical value $v_F \approx 10^6 \text{ ms}^{-1}$ and $\Upsilon_0 \approx \Delta_0/k_B$

$$\Upsilon_0^{min} \approx 10 \text{ K}, \quad (146)$$

which is the order of magnitude expected from the experiment. In Figs. 31 and 32 the considerations made are depicted. Unfortunately, the scatter of experimen-

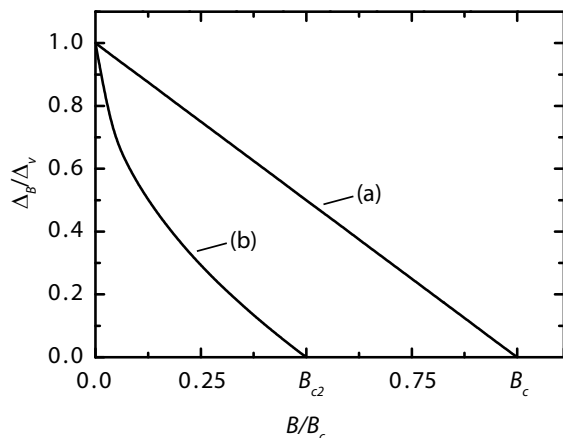


FIG. 31: Negative virtual energy Δ_B to its value Δ_v at $B = 0$ vs B/B_c . Electrons have two options to interact with each other and choose the one with which they can maximize their binding energy Δ_B . In the first scenario (a), Δ_B is given by (121). In the second scenario (b), Δ_B is given by (142). Since (121) exceeds (142) for all fields, electrons only exhibit the first scenario and we have a type I superconductor.

tal values of B_{c1} and B_{c2} is great which is why they are not compared with the theory. At last, the magnetic properties of a type II superconductor are discussed. For $B < B_{c1}$, the behavior of the electrons is equivalent to the one in a type I superconductor. For $B_{c1} < B < B_{c2}$, though, a different behavior occurs for a part of them. Since in this scenario the electron is moving in a circular

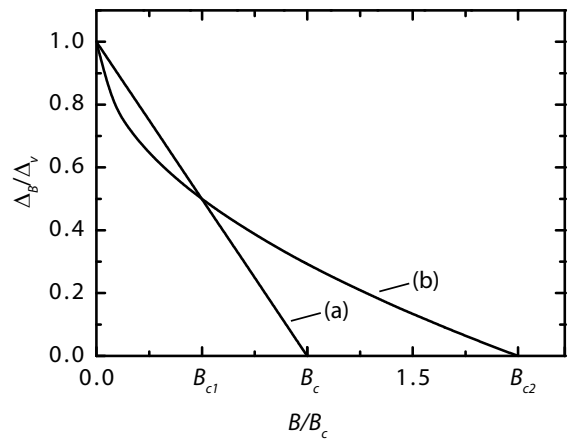


FIG. 32: Negative virtual energy Δ_B to its value Δ_v at $B = 0$ vs B/B_c . Electrons have two options to interact with each other and choose the one with which they can maximize their binding energy Δ_B . In the first scenario (a), Δ_B is given by (121). In the second scenario (b), Δ_B is given by (142). For $B < B_{c1}$, (121) exceeds (142) and electrons exhibit the first scenario. For $B_{c1} < B < B_{c2}$, (142) exceeds (121) and electrons exhibit the second scenario. Because electrons exhibit both scenarios we have a type II superconductor.

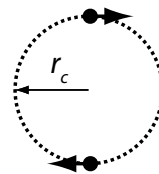


FIG. 33: Two interacting electrons in the second scenario. The radius of the circular orbit in which they are moving equals $r_c = \sqrt{\hbar/eB}$. During one run the virtual relaxation processes are performed and, thereafter, the inverse ones are performed during a second run.

orbit, the configuration of the two electrons interacting with each other is expected to consist of one orbit in which both are moving on the opposite side, see Fig. 33. As it can be shown,²⁴ an electron moving in a closed orbit is connected with a magnetic flux of a flux quantum

$$\Phi_0 = \frac{h}{e}, \quad (147)$$

or, in our case, two electrons moving in a closed orbit are connected with a flux quantum

$$\Phi_0 = \frac{h}{2e}. \quad (148)$$

Therefore, as it is observed, the magnetic field is penetrating into the sample because every bond in the second scenario leads to the penetration of the magnetic flux of $\Phi_0 = h/2e$. In the remaining part, again, it is expelled out of. At the end, I note that the critical fields B_c , B_{c1} ,

and B_{c2} given by (122), (135), and (140), respectively, fulfill the condition

$$B_c^2 = B_{c1}B_{c2}. \quad (149)$$

VI. CONCLUSION

This work approaches the mystery of understanding superconductivity from a new direction. Instead of using the proven quantum physical equations, it is tried to get to the origins of interactions, which were so far not questioned. Therefore, a new idea is postulated which is that particles may possess an additional excitation energy which is called virtual. Its magnitude is determined by equating one central equation of quantum physics, Heisenberg's uncertainty relation $t(E) = 2\hbar/E$ and the central equation for the electrical resistivity, the Drude formula $t(E) = m/\rho(E)ne^2$, where it turns out that the obtained orders of magnitude are appropriate. And, with the aid of the virtual energy, particles have energetic leeway which they use to interact with each other. In the following, I sum up my obtained results.

At first, I emphasize that with the aid of the quantities concerning time, energy, space, and momentum obtained from the equating all major phenomena connected with superconductivity can be explained, where which of and in which way they had to be used imposed itself. At this point, however, it should be said that there exists a significant difference between the two treated solid-state configurations. For the normal metals, the behavior of and the properties of the electrons can be assumed to be well understood, e.g., we know their states and the excitation-energy-dependent relaxation time. For the antiferromagnetic lattice, though, this understanding is not so well-developed, e.g., there is a lack of an established excitation-energy-dependent relaxation time. Therefore, to illustrate its functioning, the postulate is mostly applied to the normal metals. Furthermore, as one can convince oneself from Subsecs. V A to V E, the new approach simplifies the description of superconductivity. Now, I would like to emphasize the two strengths of this work. First, it sets superconducting in relation to other properties of a solid, e.g., electrical via the relaxation time and thermal via the equipartition relation, which makes superconductivity no longer as isolated. Second, the in my opinion decisive criterion for a good theory, it makes predictions in good agreement with the experiment. The most impressive ones are: The formulas for the critical magnetic fields [B_c given by (122), B_{c1} by (135), and B_{c2} by (140)] and the condition for type II superconductivity given by (145), where all expressions only contain well-known quantities. And, the temperature dependences Δ_v given by (75), c_e by (106), and B_c by (124). All these expressions have, due to their simplicity and precision, the quality to replace the so far established ones. This is why this work makes in any case an important contribution to superconductivity.

At the end, I note that, if the presented ideas turn out to be true, they improve our understanding of physics on a grand scale and they would open up a new door to do physics. And, if this case arises, I hope that this work awakes enthusiasm for everyone who enjoys exploring new and unknown territory.

APPENDIX A: INFLUENCE OF CONJURED UP VIRTUAL PARTICLES ON RELAXATION PROCESSES

To obtain the influence, I look how many more hops are needed on average to lose the excitation energy as a function of P_{gg} . Hence, I calculate after how many hops on average the energy Ξ_x is lost which leads to multiplying functions (32) and (37) by a factor W . If no virtual particles are present, $P_{gg} = 1$, it is lost after one hop. If they are present, it is lost after one hop with the probability

$$P_{gg}, \quad (A1)$$

after two hops with the probability

$$P_{gg}(1 - P_{gg}), \quad (A2)$$

after three hops with the probability

$$P_{gg}(1 - P_{gg})^2, \quad (A3)$$

and after i hops with the probability

$$P_{gg}(1 - P_{gg})^{i-1}. \quad (A4)$$

$(1 - P_{gg})^{i-1}$ is the probability of finding $i - 1$ hops that cost no energy which is the condition for losing Ξ_x after i hops. Now, W is obtained by adding up all probabilities and weighting each with its number of hops,

$$W = \sum_{i=1}^{\infty} iP_{gg}(1 - P_{gg})^{i-1}. \quad (A5)$$

Furthermore, multiplying (A5) by $1 - P_{gg}$ yields

$$W(1 - P_{gg}) = \sum_{i=1}^{\infty} iP_{gg}(1 - P_{gg})^i. \quad (A6)$$

Next, (A6) is subtracted from (A5)

$$WP_{gg} = \sum_{i=1}^{\infty} P_{gg}(1 - P_{gg})^{i-1}, \quad (A7)$$

where it is assumed that the last term of the summation in (A6) vanishes,

$$\lim_{i \rightarrow \infty} iP_{gg}(1 - P_{gg})^i = 0, \quad (\text{A8})$$

where it should be kept in mind that P_{gg} and $1 - P_{gg}$ are probabilities and take on values $0 \leq P_{gg}, 1 - P_{gg} \leq 1$. Furthermore, (A7) is simplified to

$$W = \sum_{i=1}^{\infty} (1 - P_{gg})^{i-1}. \quad (\text{A9})$$

The expression on the right-hand side of (A9) is equivalent to a geometric series

$$\sum_{i=1}^{\infty} (1 - P_{gg})^{i-1} = \frac{1}{P_{gg}}. \quad (\text{A10})$$

Therefore, the factor W by which function (37) has to be multiplied becomes

$$W = \frac{1}{P_{gg}}. \quad (\text{A11})$$

APPENDIX B: NEW INSIGHTS TO THE ELECTRICAL RESISTIVITY

The electrical resistivity can be described by the Drude model. In it the formula for the resistivity is

$$\rho = \frac{m}{ne^2t}, \quad (\text{B1})$$

where m is the mass and e the charge of an electron, which are constants, n is the density of electrons, which is a constant (semiconductors are an exception because in them n depends on the temperature), and t the relaxation time. To my mind, the quantities m , e , and n are understood. However, for the relaxation time I think this is not the case, where I understand it as follows: The relaxation time t is the mean period of time within which an electron loses its excitation energy E , where E is the total excitation energy resulting from all mechanisms responsible for a gain of energy.²⁵ From the new interpretation new insights follow, such as, an explanation for Ohm's law and a new limit for its range of validity, which are presented next by means of electrons in a normal metal.

At first, the excitation energy of an electron is expressed as

$$E = E_{therm} + E_{el}, \quad (\text{B2})$$

where E_{therm} is the thermal and E_{el} the electric energy gained due to the applied electric field. Then, E_{therm} is connected with T via

$$E_{therm} = \frac{3}{2}k_B T. \quad (\text{B3})$$

Next, E_{el} is estimated with the drift velocity v_D and the electric current density

$$j = \frac{1}{\rho} \mathcal{E} \quad (\text{B4})$$

$$= nev_D, \quad (\text{B5})$$

where $\mathcal{E}$ is the electric field. From (B4) and (B5) one gets

$$v_D = \frac{1}{\rho ne} \mathcal{E}. \quad (\text{B6})$$

Furthermore, Eq. (44) is used to connect v_D with E_{el}

$$E_{el} = \frac{1}{2}m(2v_F v_D \cos \gamma + v_D^2), \quad (\text{B7})$$

where $p = mv$ is used and v_v is identified with v_D . Expression (B7) is further reduced to

$$E_{el} = mv_F v_D, \quad (\text{B8})$$

because it is expected that $v_F \gg v_D$ and therefore electrons with $\cos \gamma = 1$ dominate the magnitude of E_{el} .²⁶ Now, by inserting (B6) into (B8) one obtains

$$E_{el} = \frac{mv_F}{\rho ne} \mathcal{E}. \quad (\text{B9})$$

Next, the energies E_{therm} and E_{el} are compared. To do that $\rho(E)$ is obtained by combining (B1), (21), and (22)

$$\rho(E) = \frac{124m}{ne^2 A} \left(\frac{2E}{3k_B \Theta} \right)^5, \quad (\text{B10})$$

where it is assumed that thermal and electric processes can be treated in a related way so that the resistivity can be described by (B10) in which E is given by (B2). Now, the condition for $E_{therm} = E_{el}$ is determined to be able to say which energy dominates. Since $E_{therm} = E_{el}$ implies that $E = E_{therm} + E_{el} = 2E_{therm} = 3k_B T$, equating (B3) and (B9) and using (B10) yields

$$\frac{T^6}{\mathcal{E}} = \frac{ev_F A \Theta^5}{5952k_B}. \quad (\text{B11})$$

In case that

$$\frac{T^6}{\mathcal{E}} \gg \frac{ev_F A \Theta^5}{5952k_B} \quad (\text{B12})$$

one gets $E_{therm} \gg E_{el}$, and in case that

$$\frac{T^6}{\mathcal{E}} \ll \frac{ev_F A \Theta^5}{5952k_B} \quad (\text{B13})$$

one gets $E_{therm} \ll E_{el}$. In addition, (B11) is rewritten to

$$T_{Ohm} = \left(\frac{ev_F A \Theta^5}{5952k_B} \mathcal{E} \right)^{\frac{1}{6}}, \quad (\text{B14})$$

$$\mathcal{E}_{Ohm} = \frac{5952k_B}{ev_F A \Theta^5} T^6, \quad (\text{B15})$$

where the temperature at which $E_{therm} = E_{el}$ for the electric field $\mathcal{E}$ is called Ohm's temperature T_{Ohm} , and the electric field for which $E_{therm} = E_{el}$ at the temperature T Ohm's field $\mathcal{E}_{Ohm}$. To get a feeling of the orders of magnitude, values of $A\Theta^5$ and v_F from Tables II and V for aluminum and the typical values $\mathcal{E} \approx 1 \text{ Vm}^{-1}$ and $T \approx 300 \text{ K}$ are inserted into (B14) and (B15)

$$T_{Ohm}^{\text{Al}} \approx 10 \text{ K}, \quad (\text{B16})$$

$$\mathcal{E}_{Ohm}^{\text{Al}} \approx 10^{10} \frac{\text{V}}{\text{m}}. \quad (\text{B17})$$

At $T \gg T_{Ohm}$ and for $\mathcal{E} \ll \mathcal{E}_{Ohm}$ one gets $E_{therm} \gg E_{el}$, and at $T \ll T_{Ohm}$ and for $\mathcal{E} \gg \mathcal{E}_{Ohm}$ one gets $E_{therm} \ll E_{el}$. This behavior accounts for the validity of Ohm's law because, usually, one examines it at $T \gg T_{Ohm}$ and for $\mathcal{E} \ll \mathcal{E}_{Ohm}$ (e.g., at room temperature $T \approx 300 \text{ K}$ and for $\mathcal{E} \approx 1 \text{ Vm}^{-1}$), where $E_{therm} \gg E_{el}$. This means that the excitation energy becomes $E = E_{therm}$ because E_{el} in (B2) can be neglected and that the relaxation time becomes independent of $\mathcal{E}$. Therefore, because m , e , n , and t are constant values, the electrical resistivity (B1) becomes a constant value which is the statement of Ohm's law. Furthermore, there exists the case $T \ll T_{Ohm}$ and $\mathcal{E} \gg \mathcal{E}_{Ohm}$ in which Ohm's law is not valid since ρ depends on $\mathcal{E}$. Now, I examine this limit $E_{therm} \ll E_{el}$ which means that $E = E_{el}$. Hence, E_{el} given by (B8) is inserted into (B10), and thereafter this expression is inserted into (B4)

$$j = \frac{ne^2 A}{124m} \left(\frac{3k_B \Theta}{2mv_F v_D} \right)^5 \mathcal{E}. \quad (\text{B18})$$

Next, v_D given by $v_D = j/ne$ is inserted into (B18)

$$j = \left(\frac{n^6 e^7 k_B^5 A \Theta^5}{16m^6 v_F^5} \right)^{\frac{1}{6}} \mathcal{E}^{\frac{1}{6}}. \quad (\text{B19})$$

As can be seen in the limit $E_{therm} \ll E_{el}$ the linear connection between j and $\mathcal{E}$ is not valid. In Figs. 34 and 35 the considerations made are illustrated. One insight

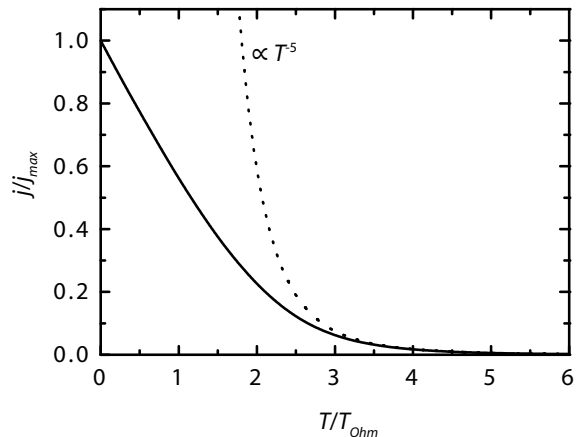


FIG. 34: The electric current density j to its maximum value j_{max} at $T = 0 \text{ K}$ vs T/T_{Ohm} for a constant electric field. The relationship between j and T is obtained by inserting (B10) into (B4) and using $E = E_{therm} + E_{el} = 3k_B T/2 + mv_F j/ne$. The values of v_F , n , and $A\Theta^5$ are taken from Tables II and V for aluminum and $\mathcal{E}$ is chosen to be $\mathcal{E} = 1 \text{ Vm}^{-1}$. The dashed line represents the picture developed so far in which the energy E is only identified with the thermal energy $E = 3k_B T/2$. In the limit $T \gg T_{Ohm}$ ($E_{therm} \gg E_{el}$) the lines coincide.

that also follows is that the picture that the electrical resistivity in a perfect crystalline solid vanishes if the temperature vanishes is false (i.e., $\lim_{T \rightarrow 0} \rho > 0$ instead of $\lim_{T \rightarrow 0} \rho = 0$). This is so because even if the temperature becomes small or vanishes, there still exists E_{el} which leads to a finite resistivity, where E_{el} vanishes only if $\mathcal{E} = 0$ when no electric current flows. And, for a given electric field $\mathcal{E}$, there exists the maximum current density j_{max} at $T = 0 \text{ K}$ given by (B19).

At last, I present an explanation for the temperature independence of ρ due to scattering on lattice imperfections. It is assumed that their density is independent of the temperature and that the scattering probability and thereby the resistivity is proportional to the velocity of the electrons

$$\rho \propto v, \quad (\text{B20})$$

which is inspired by the picture that the faster an electron moves, the more often it passes a lattice imperfection on which it is scattered. Its velocity v constitutes of the Fermi velocity v_F , the thermal velocity v_{therm} , and the drift velocity v_D . Since it is expected that $v_F \gg v_{therm}, v_D$,²⁷ one gets $v = v_F$ which means that v and ρ are independent of v_{therm} and v_D , and thereby of the temperature T and the electric field $\mathcal{E}$.

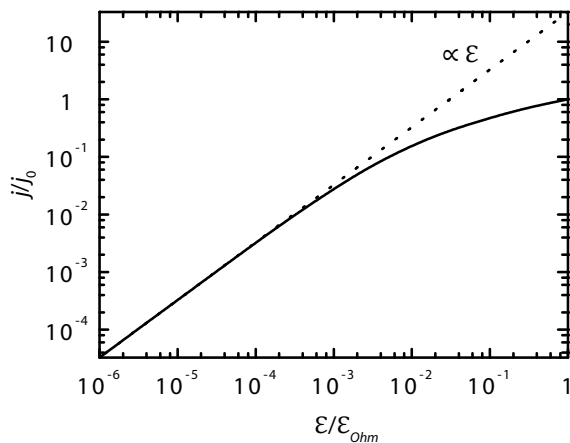


FIG. 35: The electric current density j to its value j_0 at $\mathcal{E}/\mathcal{E}_{Ohm} = 1$ vs $\mathcal{E}/\mathcal{E}_{Ohm}$ at a constant temperature on a double-logarithmic scale. The relationship between j and $\mathcal{E}$ is obtained by inserting (B10) into (B4) and using $E = E_{therm} + E_{el} = 3k_B T/2 + mv_F j/ne$. The values of v_F , n , and $A\Theta^5$ are taken from Tables II and V for aluminum and T is chosen to be $T = 300$ K. The dashed line represents Ohm's law in the picture developed so far in which the energy E is only identified with the thermal energy $E = 3k_B T/2$. The picture developed within this work says that Ohm's law holds only in the limit $\mathcal{E} \ll \mathcal{E}_{Ohm}$ ($E_{therm} \gg E_{el}$) and that the resistivity depends on $\mathcal{E}$ as soon as $\mathcal{E}$ reaches the order of magnitude of $\mathcal{E}_{Ohm}$.

* Electronic address: contact@buttler-physics.com

¹ H. Kamerlingh Onnes, Leiden Comm. **120b**, **122b**, **124c** (1911).

² J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).

³ J. G. Bednorz and K. A. Müller, Z. Phys. **B64**, 189 (1986).

⁴ By the “heart of superconductivity” I mean the central thought on which all explanations are based.

⁵ R. P. Feynman, Rev. Mod. Phys. **29**, 205 (1957).

⁶ The expression “in the picture developed so far” is used at several points. However, I am not going to give literature references at them because, if the expression is used, it is used in a general way so that it is difficult to choose one reference over the other.

⁷ Usually, $t(E)$ is written as the temperature-dependent relaxation time $t(T)$. But since T corresponds to an energy E via the equipartition relation, both functions can be converted into each other.

⁸ It is noted that Debye temperatures obtained from the specific heat of phonons lie between 100 K and 400 K (see reference in Table II). These values are too small to fulfill $50 \text{ K} \ll \Theta$ very well. Nevertheless, the assumption is made because it simplifies the treatment.

⁹ A treatment of this subject can be found in *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. XIV, and *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. XIX.

¹⁰ The last assumption resembles Fermat's principle of taking the path that can be passed in a minimum amount of time.

¹¹ The depiction in Figs. 14 and 15 may thus be misleading because it suggests that the charge carriers possess well-defined energies. Actually, a charge carrier could not even perform a hop that costs energy if it would not be in an energy band in which it can exhibit an excitation energy.

¹² The fact that the phenomenon of superconductivity is attributable to the emergence of a new interaction should be emphasized because it distinguishes superconductivity from all other phenomena in a solid which can be described by the electromagnetic interaction, i.e., the virtual particle whose creation or annihilation is responsible for the change of the state of motion is a photon instead of the particle created in a relaxation process, e.g., a phonon in normal metals.

¹³ An interaction between more than two, e.g., three electrons, could be: Electron a loses E_v^a and gains E_v^b . Electron b loses E_v^b and gains E_v^c . Electron c loses E_v^c and gains E_v^a .

¹⁴ It is expected that W decreases Δ_v and π_v by the same amount as it increases τ_v and ξ_v so that Eqs. (51) and (52) hold.

¹⁵ The assumption $\Theta \approx \text{const}$ means that the Debye temperature Θ_{el} in the resistivity cannot be set equal to the Debye temperature Θ_{ph} in the specific heat of phonons because Θ_{ph} can be quite different (see reference in Table II).

¹⁶ The relationship between E_0 and T_0 (Δ_0 and Υ_0) is examined in Subsec. V C.

¹⁷ Theoretical views and experimental results to this subject can be found in *Handbook of High-Temperature Superconductivity: Theory and Experiment*, J. R. Schrieffer and J.

- S. Brooks (Springer-Verlag, Berlin, 2007).
- ¹⁸ Typical temperatures Υ_0 are at maximum of the order of 10 K which fulfill the assumption $T \lesssim 10 \text{ K} \ll \Theta$ in (20) well, which is why the temperature dependence (75) is expected to hold well.
- ¹⁹ In case that E is not proportional to T , (84) must be re-expressed.
- ²⁰ One review of experimental data is given in J. Y. T. Wei, C. C. Tsuei, P. J. M. van Bentum, Q. Xiong, C. W. Chu, and M. K. Wu, Phys. Rev. B **57**, 3650 (1998). Within it Δ_0/Υ_0 takes on values $2k_B \lesssim \Delta_0/\Upsilon_0 \lesssim 5k_B$. The deviations from $1.5k_B$ could be explained by taking the quantum nature into account when determining the relationship between E and T , i.e., the relationship between Δ_0 and Υ_0 .
- ²¹ A derivation of the expression can be found in *Festkörperphysik*, H. Ibach and H. Lüth (Springer-Verlag, Berlin, 2002).
- ²² W. S. Corak, B. B. Goodman, C. B. Satterthwaite, and A. Wexler, Phys. Rev. **96**, 1442 (1954); **102**, 656 (1956).
- ²³ Experimental data can be found in J. W. Loram, K. A. Mirza, J. R. Cooper, and W. Y. Liang, Phys. Rev. Lett. **71**, 1740 (1993).
- ²⁴ H. Ibach and H. Lüth, *Festkörperphysik* (Springer-Verlag, Berlin, 2002).
- ²⁵ So far, the energy E in $t(E)$ was only identified with the excitation energy resulting from the thermal energy and the relaxation time t with the period of time within which the electric energy is lost, i.e., the drift velocity becomes zero.
- ²⁶ The case that $v_D \approx v_F \approx 10^6 \text{ ms}^{-1}$ means that $j = nev_D \approx 10^{16} \text{ Am}^{-2}$, if $n \approx 10^{29} \text{ m}^{-3}$, which is such a high current density that it is in general not reached.
- ²⁷ The order of magnitude of v_{therm} is estimated by $v_{therm} = E_{therm}/mv_F$ [cf. Eq. (B8)] which yields at $T \approx 300 \text{ K}$ and for $v_F \approx 10^6 \text{ ms}^{-1}$ the value $v_{therm} \approx 10^4 \text{ ms}^{-1} \ll v_F$. The order of magnitude of v_D is estimated in footnote 26.